

# Stabilizing Transmission Intervals and Delays in Nonlinear Networked Control Systems through Hybrid-System-with-Memory Modeling and Lyapunov-Krasovskii Arguments

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- 2 Problem Formulation
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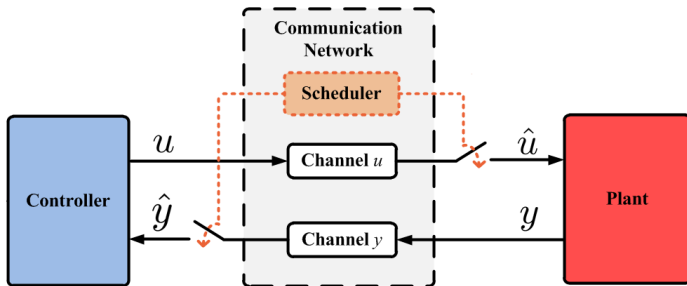
# From Analog to Digital and Networked World

- many processes exhibit both **continuous** and **discrete** dynamics – **hybrid dynamics**<sup>[1]</sup>
- colliding rigid bodies and electrical circuits with active components
- **digital technology** for monitoring and controlling purposes
- communication/measuring devices, sampling units and logic devices
- **delays** in control processes per se and owing to digital technology

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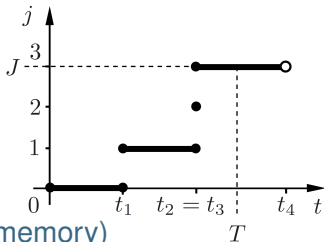
[1] Rafal Goebel, Ricardo G. Sanfelice, and Andrew R. Teel. *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*. Princeton University Press, 2012.

# Real-Life Control System

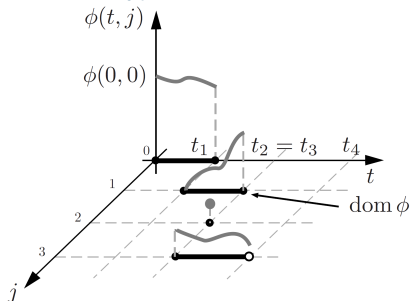


# Hybrid Systems with Memory

- hybrid time domain (with memory)



- hybrid arc (with memory)



# Hybrid Systems with Memory

- given  $d \geq 0$ , we use  $\mathcal{M}^d$  to denote the **collection of hybrid memory arcs**  $\phi$  satisfying  $s \geq -d$  for all  $(s, k) \in \text{dom } \phi$
- system state**  $x_t : \text{dom}_{\geq 0}(x) \rightarrow \mathcal{M}^d$  defined by

$$x_t(s, k(s)) := x(t + s, j + k(s))$$

for all  $(s, k(s)) \in \text{dom } x_t$ , where

$$k(s) := \max\{k : (s, k) \in \text{dom } x_t\} \text{ and } \text{dom } x_t := \{(s, k) \in \mathbb{R}_{\leq 0} \times \mathbb{Z}_{\leq 0} : (t + s, j + k) \in \text{dom } x, s \in [-d, 0]\}$$

- $d$  is the **maximum value** of all delay phenomena in the dynamics

# Hybrid Systems with Memory

## Definition

A **hybrid system with memory**  $d$  is defined by a 4-tuple  $\mathcal{H}_{\mathcal{M}}^d = (\mathcal{C}, \mathcal{F}, \mathcal{D}, \mathcal{G})$  as follows:

- a set  $\mathcal{C} \subset \mathcal{M}^d$ , called the **flow set**,
- a function  $\mathcal{F} : \mathcal{M}^d \rightarrow \mathbb{R}^{n_x}$ , called the **flow map**,
- a set  $\mathcal{D} \subset \mathcal{M}^d$ , called the **jump set**, and
- a function  $\mathcal{G} : \mathcal{M}^d \rightarrow \mathbb{R}^{n_x}$ , called the **jump map**.

[2] Jun Liu and Andrew R. Teel. "Lyapunov-Based Sufficient Conditions for Stability of Hybrid Systems With Memory". In: *IEEE Trans. on Automatic Control* 61.4 (2016), pp. 1057–1062.

# Hybrid Systems with Memory

## Definition

A hybrid arc is a **solution** to the hybrid system  $\mathcal{H}_{\mathcal{M}}^d$  if the **initial data**  $x_0 \in \mathcal{C} \cup \mathcal{D}$  and

- (i) for all  $j \in \mathbb{Z}_{\geq 0}$  and almost all  $t$  such that  $(t, j) \in \text{dom}_{\geq 0}(x)$

$$x_t \in \mathcal{C}, \quad \dot{x}(t, j) = \mathcal{F}(x_t),$$

- (ii) for all  $(t, j) \in \text{dom}_{\geq 0}(x)$  such that  $(t, j + 1) \in \text{dom}_{\geq 0}(x)$ ,

$$x_t \in \mathcal{D}, \quad x(t, j + 1) = \mathcal{G}(x_t).$$



# Set pre-UGAS

## Definition

Let  $\mathcal{W} \in \mathbb{R}^{n_x}$  be a **closed set**. The set  $\mathcal{W}$  is **Uniformly Globally pre-Asymptotically Stable** (UGpAS) for system  $\mathcal{H}_{\mathcal{M}}^d$  if there exists a  $\mathcal{KL}$  function  $\beta$  such that any solution  $x$  to  $\mathcal{H}_{\mathcal{M}}^d$  satisfies

$$\|x(t, j)\|_{\mathcal{W}} \leq \beta\left(\|x_0\|_{\mathcal{W}}, t + j\right), \quad \forall (t, j) \in \text{dom}_{\geq 0}(x),$$

where  $\|z\|_{\mathcal{W}} := \inf_{y \in \mathcal{W}} \|y - z\|$  for  $z \in \mathbb{R}^{n_x}$  and

$$\|\phi\|_{\mathcal{W}} := \sup_{\substack{(s, k) \in \text{dom } \phi \\ s \in [-d, 0]}} \|\phi(s, k)\|_{\mathcal{W}} \text{ for } \phi \in \mathcal{M}^d.$$

# Contributions

- 1 hybrid systems-with-memory modeling for a large class of NCSs

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[3] Domagoj Tolić. “Stabilizing transmission intervals and delays in nonlinear networked control systems through hybrid-system-with-memory modeling and Lyapunov–Krasovskii arguments”. In: *Nonlinear Analysis: Hybrid Systems* 36 (2020), p. 100834.

# Contributions

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- 2 more general and less conservative sufficient conditions for finding stabilizing transmission intervals and delays in nonlinear NCSs

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# Contributions

- 1 hybrid systems-with-memory modeling for a large class of NCSs
- 2 more general and less conservative sufficient conditions for finding stabilizing transmission intervals and delays in nonlinear NCSs
- 3 the first Lyapunov-based small-gain theorem for hybrid systems with memory
- 4 novel robustness analyses for LTI<sup>[3]</sup> control systems involving plant input delays, output feedback, observer-predictor-based dynamic controllers and network-induced distributed delays

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[3] Domagoj Tolić. “Stabilizing transmission intervals and delays in nonlinear networked control systems through hybrid-system-with-memory modeling and Lyapunov–Krasovskii arguments”. In: *Nonlinear Analysis: Hybrid Systems* 36 (2020), p. 100834.

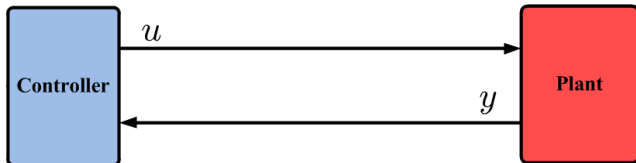
# Stabilizing Transmission Intervals and Delays

- nonlinear delayed **plant**

$$\begin{aligned}\dot{x}_p &= f_p(t, x_{p_t}, u_t), \\ y &= g_p(t, x_{p_t}),\end{aligned}\tag{1}$$

- nonlinear delayed **controller**

$$\begin{aligned}\dot{x}_c &= f_c(t, x_{c_t}, y_t), \\ u &= g_c(t, x_{c_t}),\end{aligned}\tag{2}$$



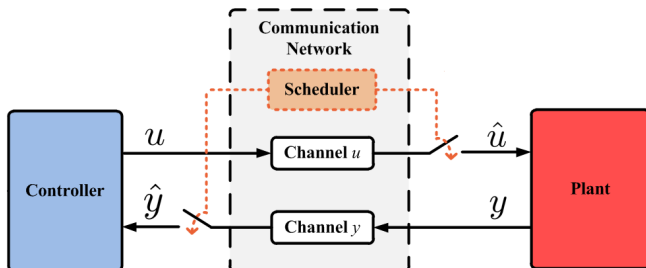
# Introduce Communication Network

- $\ell$  communication channels with **discrete delays**

$$(y(t - d_i(t)), u(t - d_i(t)))_i, \quad i \in \{1, \dots, \ell\} \quad (3)$$

or **distributed delays**

$$\int_{t-d_i(t)}^t \Gamma_i(t, s, (y(s), u(s)))_i ds, \quad i \in \{1, \dots, \ell\} \quad (4)$$



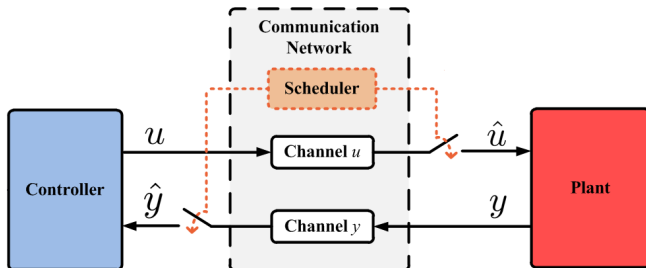
# Introduce Communication Network

- nonlinear delayed plant

$$\begin{aligned}\dot{x}_p &= f_p(t, x_{p_t}, \hat{u}_t), \\ y &= g_p(t, x_{p_t}),\end{aligned}\quad (5)$$

- nonlinear delayed controller

$$\begin{aligned}\dot{x}_c &= f_c(t, x_{c_t}, \hat{y}_t), \\ u &= g_c(t, x_{c_t}).\end{aligned}\quad (6)$$



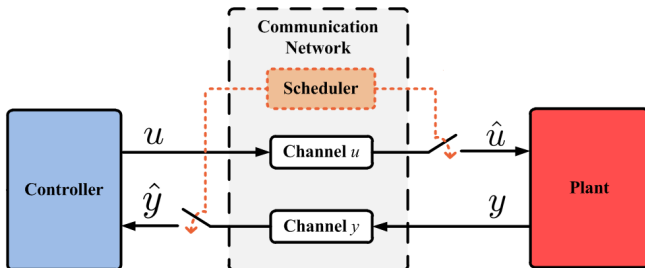


# Stabilizing Transmission Intervals and Delays

- define the **error vector**

$$e(t) = \begin{bmatrix} e_y(t) \\ e_u(t) \end{bmatrix} := \begin{bmatrix} \hat{y}(t) - y^*(t) \\ \hat{u}(t) - u^*(t) \end{bmatrix}, \quad (7)$$

where the components of  $(y^*(t), u^*(t))$  are given by (3)-(4)



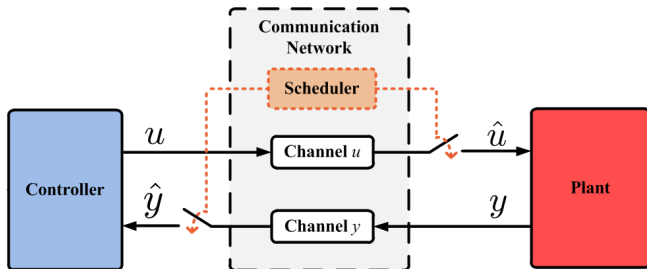
# Stabilizing Transmission Intervals and Delays

- $\hat{y}$  and  $\hat{u}$  are updated at instants  $t_\kappa$ ,  $\kappa \in \mathbb{Z}_{\geq 0}$ , such that  $0 < \epsilon \leq t_{\kappa+1} - t_\kappa \leq \tau_{\text{MATI}}$ , yielding

$$\begin{aligned}\hat{y}(t_\kappa^+) &= y^*(t_\kappa) + h_y(\kappa, e(t_\kappa)), \\ \hat{u}(t_\kappa^+) &= u^*(t_\kappa) + h_u(\kappa, e(t_\kappa)),\end{aligned}\tag{8}$$

- consequently

$$e_i(t_\kappa^+) = \mathbf{0}_{n_{e_i}}\tag{9}$$



# Hybrid Systems with Memory

- altogether<sup>[4]</sup>:

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{e} \\ \dot{\tau}_1 \\ \dot{\kappa} \\ \dot{\tau}_2 \end{bmatrix} = \begin{bmatrix} f(x_t, e_t, \tau_{2_t}) \\ g(x_t, e_t, \tau_{2_t}) \\ 1 \\ 0 \\ 1 \end{bmatrix} = \mathcal{F}(\xi_t), \underbrace{\tau_{1_t}(0,0) \in [0, \tau_{MATI}]}_C, \quad (10)$$

$$\xi^+ = \begin{bmatrix} x^+ \\ e^+ \\ \tau_1^+ \\ \kappa^+ \\ \tau_2^+ \end{bmatrix} = \begin{bmatrix} x_t(0,0) \\ h(\kappa_t(0,0), e_t(0,0)) \\ 0 \\ \kappa_t(0,0) + 1 \\ \tau_{2_t}(0,0) \end{bmatrix} = \mathcal{G}(\xi_t), \underbrace{\tau_{1_t}(0,0) \in [\epsilon, \tau_{MATI}]}_D, \quad (11)$$

where  $\xi := (x, e, \tau_1, \kappa, \tau_2)$

<sup>[4]</sup> Contribution 1

# Hybrid Systems with Memory

$$f(x_t, e_t, \tau_{2_t}) \stackrel{(5),(6)}{:=} \begin{bmatrix} f_p(\tau_{2_t}, x_{p_t}, \underbrace{g_{c_t}(\tau_{2_t}, x_{c_t}) + e_{u_t}}_{=\hat{u}_t \text{ using (6) and (7)}}) \\ f_c(\tau_{2_t}, x_{c_t}, \underbrace{g_{p_t}(\tau_{2_t}, x_{p_t}) + e_{y_t}}_{=\hat{y}_t \text{ using (5) and (7)}}) \end{bmatrix} \quad (12)$$

$$h(\kappa_t(0, 0), e_t(0, 0)) := \begin{bmatrix} h_y(\kappa_t(0, 0), e_t(0, 0)) \\ h_u(\kappa_t(0, 0), e_t(0, 0)) \end{bmatrix} \quad (13)$$

$$g(x_t, e_t, \tau_{2_t}) \stackrel{(7)}{:=} \begin{bmatrix} \underbrace{\hat{f}_p(\tau_{2_t}, x_{p_t}, x_{c_t}, g_{p_t}(x_{p_t}) + e_{y_t}, g_{c_t}(x_{c_t}) + e_{u_t})}_{\text{model-based estimator}} - \dot{y}^*(\tau_{2_t}(0, 0)) \\ \underbrace{\hat{f}_c(\tau_{2_t}, x_{p_t}, x_{c_t}, g_{p_t}(x_{p_t}) + e_{y_t}, g_{c_t}(x_{c_t}) + e_{u_t})}_{\text{model-based estimator}} - \dot{u}^*(\tau_{2_t}(0, 0)) \end{bmatrix} \quad (14)$$

# Stabilizing Transmission Intervals and Delays

## Problem

Find  $\tau_{\text{MATI}}$  that renders the **set**  $\{(x, e, \tau_1, \kappa, \tau_2) : x = \mathbf{0}_{n_x}, e = \mathbf{0}_{n_e}\}$  **UGpAS** for the **closed-loop** hybrid dynamics with memory (10)-(11).

# Assumptions

## Definition

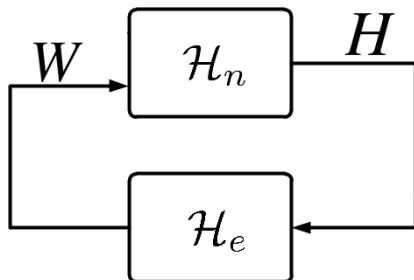
The **protocol** given by  $h := (h_y, h_u)$  is **UGES** if there exist a function  $W : \mathbb{Z}_{\geq 0} \times \mathbb{R}^{n_e} \rightarrow \mathbb{R}_{\geq 0}$  that is locally Lipschitz in its second argument, real number  $\rho \in [0, 1)$  and functions  $\underline{\alpha}$ ,  $\bar{\alpha} \in \mathcal{K}_\infty$  such that the following holds for all  $\kappa \in \mathbb{Z}_{\geq 0}$  and  $e \in \mathbb{R}^{n_e}$ :

$$\underline{\alpha}(\|e\|) \leq W(\kappa, e) \leq \bar{\alpha}(\|e\|), \quad (15)$$

$$W(\kappa + 1, h(\kappa, e)) \leq \rho W(\kappa, e), \quad (16)$$

Common UGES protocols are Round Robin (RR) and Try Once Discard (TOD).

# Inferring Stability from This Interconnection



# Assumptions

## Assumption

There exist a locally Lipschitz **Lyapunov-Krasovskii functional**  $V : \mathcal{M}_{n_x}^d \rightarrow \mathbb{R}_{\geq 0}$ , a continuous functional  $H : \mathcal{M}_{n_x}^d \rightarrow \mathbb{R}_{\geq 0}$ , real numbers  $L_i, J_j \geq 0$ , and a continuous positive-definite function  $\varrho : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  such that:

- (i) for all  $e \in \mathbb{R}^{n_e}$ ,  $\kappa \in \mathbb{Z}_{\geq 0}$ , and  $x_t \in \mathcal{M}_{n_x}^d$  the inequality

$$\dot{V}(x_t, \dot{x}_t) \leq -\varrho(\|x_t(0, 0)\|) - H^2(x_t) + \gamma^2 W^2(\kappa, e) \quad (17)$$

holds almost everywhere along the solutions of the **nominal system**  $\mathcal{H}_n$ , and

- (ii) for all nonnegative hybrid memory arcs  $\kappa_t \in \mathcal{M}_{n_\kappa}^d$ , all  $x_t \in \mathcal{M}_{n_x}^d$  and  $e_t \in \mathcal{M}_{n_e}^d$  the inequality (18) holds almost everywhere along the solutions of the **error system**  $\mathcal{H}_e$ , where  $\Delta_i(t)$ 's and  $\delta_j(t)$ 's are differentiable satisfying  $\Delta_i(t) \leq \bar{\Delta}_i$ ,  $|\dot{\Delta}_i(t)| \leq \check{\Delta}_i < 1$ ,  $\delta_j(t) \leq \bar{\delta}_j$  and  $|\dot{\delta}_j(t)| \leq \check{\delta}_j < 1$ .



# Assumptions

- the error dynamics upper bound<sup>[5]</sup>:

$$\begin{aligned}
 & \left\langle \frac{\partial W(\kappa_t(0, 0), e_t(0, 0))}{\partial e}, g(x_t, e_t, \tau_{2t}) \right\rangle \leq L_0 W(\kappa_t(0, 0), e_t(0, 0)) + \\
 & + \sum_{i=1}^n L_i W(\kappa_t(-\Delta_i(t), k(-\Delta_i(t))), e_t(-\Delta_i(t), k(-\Delta_i(t)))) + \\
 & + \sum_{j=1}^m J_j \int_{t-\delta_j(t)}^t W(\kappa_t(s, k(s)), e_t(s, k(s))) ds + H(x_t) \quad (18)
 \end{aligned}$$

[5] Contribution 

# Main Result

## Theorem

Suppose Assumption 1 is satisfied. In addition, suppose that there exist regular enough **function**  $\psi : [0, \tau_{\text{MATI}}] \rightarrow \mathbb{R}_{>0}$  and constants  $p_i, \mu_i, r_i, \nu_j \geq 0$  such that

(iii) the inequality

$$\rho^2 \leq \frac{\psi(\tau_{\text{MATI}})}{\psi(0)}, \text{ and} \quad (19)$$

(iv) the matrix inequality (20) with expressions (21)-(23) hold, then the set  $\mathcal{W} := \{(x, e, \tau_1, \kappa, \tau_2) : x = \mathbf{0}_{n_x}, e = \mathbf{0}_{n_e}\}$  is **UGPAS** for the **closed-loop** system (10)-(11).<sup>[6]</sup>

[6] Contribution **3**

# Main Result

$$\begin{bmatrix}
 \psi_0 & \psi L_1 + L_0 L_1 \Xi + r_1 & 0 & \cdots & \psi L_n + L_0 L_n \Xi + r_n & 0 & \psi J_1 + L_0 J_1 \Xi & \cdots & \psi J_m + L_0 J_m \Xi & \psi + L_0 \Xi \\
 \psi L_1 + L_0 L_1 \Xi + r_1 & \psi_1 & \frac{r_1}{\bar{\psi}_1} & \cdots & L_1 L_n \Xi & 0 & L_1 J_1 \Xi & \cdots & L_1 J_m \Xi & L_1 \Xi \\
 0 & r_1 & \bar{\psi}_1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 \psi L_n + L_0 L_n \Xi + r_n & L_1 L_n \Xi & 0 & \cdots & \psi_n & \frac{r_n}{\bar{\psi}_n} & L_n J_1 & \cdots & L_n J_m & L_n \Xi \\
 0 & 0 & 0 & \cdots & r_n & \bar{\psi}_n & 0 & \cdots & 0 & 0 \\
 \psi J_1 + L_0 J_1 \Xi & L_1 J_1 \Xi & 0 & \cdots & L_n J_1 & 0 & \Phi_1 & \cdots & J_1 J_m & J_1 \Xi \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 \psi J_m + L_0 J_m \Xi & L_1 J_m \Xi & 0 & \cdots & L_n J_m & 0 & J_1 J_m & \cdots & \Phi_m & J_m \Xi \\
 \psi + L_0 \Xi & L_1 \Xi & 0 & \cdots & L_n \Xi & 0 & J_1 \Xi & \cdots & J_m \Xi & -1 + \Xi
 \end{bmatrix} < 0 \quad (20)$$

$$\psi_0 = \gamma^2 + 2\psi L_0 + \dot{\psi} + \sum_{i=1}^n (p_i + \mu_i + r_i \bar{\Delta}_i^2 - r_i) + \sum_{j=1}^m \nu_j \bar{\delta}_j^2, \quad (21)$$

$$\psi_i = -2r_i + L_i^2 \Xi - \mu_i (1 - \check{\Delta}_i) \quad (22)$$

$$\bar{\psi}_i = -p_i - r_i, \quad \Phi_j = -\nu_j (1 - \check{\delta}_j) + J_j^2 \Xi, \quad \Xi = \sum_{i=1}^n \bar{\Delta}_i^2 r_i \quad (23)$$

# Sketch of the Proof

- define  $z(t, j) := W(\kappa(t, j), e(t, j))$  leading to

$$\begin{aligned} \dot{z}(t, j) \leq & L_0 z_t(0, 0) + \sum_{i=1}^n L_i z_t(-\Delta_i(t), k(-\Delta_i(t))) + \\ & + \sum_{j=1}^m J_j \int_{t-\delta_j(t)}^t z_t(s, k(s)) ds + H(x_t), \tau_1 \in [0, \tau_{\text{MATI}}], \end{aligned} \quad (24)$$

and

$$z(t, j+1) \leq \rho z(t, j), \quad \tau_1 \in [\epsilon, \tau_{\text{MATI}}]. \quad (25)$$

# Sketch of the Proof

- define the following Lyapunov-Krasovskii functional for  $\mathcal{H}_e$ :

$$\begin{aligned}
 V_1(z_t, \dot{z}_t, \tau_{1_t}) = & \psi(\tau_{1_t}(\mathbf{0}, \mathbf{0}))z_t^2(\mathbf{0}, \mathbf{0}) + \\
 & + \sum_{i=1}^n p_i \int_{t-\bar{\Delta}_i}^t z_t^2(s, k(s))ds + \\
 & + \sum_{i=1}^n \mu_i \int_{t-\Delta_i(t)}^t z_t^2(s, k(s))ds + \\
 & + \sum_{i=1}^n r_i \bar{\Delta}_i \int_{-\bar{\Delta}_i}^0 \int_{t+\theta}^t \dot{z}_t^2(\chi, k(\chi))d\chi d\theta + \\
 & + \sum_{j=1}^m \nu_j \bar{\delta}_j \int_{-\delta_j(t)}^0 \int_{t+\theta}^t z_t^2(\chi, k(\chi))d\chi d\theta,
 \end{aligned}$$

# Sketch of the Proof

- choose the following Lyapunov-Krasovskii functional for the **closed-loop system** (10)-(11):

$$U(\xi_t, \dot{\xi}_t) := V(x_t, \dot{x}_t) + V_1(z_t, \dot{z}_t, \tau_{1t}).$$

- owing to (19), the **jumps** are **not destabilizing** because

$$\begin{aligned} U(\xi_t^+, \dot{\xi}_t^+) &= V(x_t^+, \dot{x}_t^+) + V_1(z_t^+, \dot{z}_t^+, \tau_{1t}^+) \\ &\leq V(x_t, \dot{x}_t) + V_1(z_t, \dot{z}_t, \tau_{1t}) + [\psi(\tau_{2t}(0, 0))\rho^2 - \\ &\quad - \psi(\tau_{2t}(0, -1))]z_t^2(0, 0) \\ &= V(x_t, \dot{x}_t) + V_1(z_t, \dot{z}_t, \tau_{1t}) + [\psi(0)\rho^2 - \\ &\quad - \psi(\tau_{\text{MATI}})]z_t^2(0, 0) \leq U(\xi_t, \dot{\xi}_t). \end{aligned}$$

# Sketch of the Proof

- during **flows**, the upper Dini derivative is

$$\begin{aligned}
 D^+ U(\xi_t, \dot{\xi}_t) &\leq -\varrho(\|x_t(0,0)\|) - H^2(x_t) + \gamma^2 z_t^2(0,0) + \\
 &+ 2\psi z_t(0,0)\dot{z}_t(0,0) + \dot{\psi} z_t^2(0,0) + \\
 &+ \sum_{i=1}^n p_i (z_t^2(0,0) - z_t^2(-\bar{\Delta}_i, k(-\bar{\Delta}_i))) + \\
 &+ \sum_{i=1}^n \mu_i (z_t^2(0,0) - (1 - \check{\Delta}_i) z_t^2(-\Delta_i(t), k(-\Delta_i(t)))) + \\
 &+ \sum_{i=1}^n r_i \bar{\Delta}_i^2 z_t^2(0,0) - \sum_{i=1}^n r_i (z_t(0,0) - \\
 &- z_t(-\Delta_i(t), k(-\Delta_i(t))))^2 - \\
 &- \sum_{i=1}^n r_i (z_t(-\Delta_i(t), k(-\Delta_i(t))) - z_t(-\bar{\Delta}_i, k(-\bar{\Delta}_i)))^2 + \\
 &+ \sum_{j=1}^m \nu_j \bar{\delta}_j^2 z_t^2(0,0) - \sum_{j=1}^m \nu_j (1 - \check{\delta}_j) \left( \int_{t-\delta_j(t)}^t z_t(s, k(s)) ds \right)^2.
 \end{aligned}$$

# Main Result – Existence

## Corollary

Suppose that there exist constants  $\mu_i, \nu_j \geq 0$  such that

(M') the inequality

$$\rho^2 \leq M,$$

where  $M := e^{-\sigma \tau_{MATI}}$  and  $\sigma \geq 0$ , and

(W') the matrix inequality

$$\begin{bmatrix} N & L_1 M & \dots & L_n M & J_1 M & \dots & J_m M \\ L_1 M & -\mu_1(1-\check{\Delta}_1) & 0 & 0 & 0 & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 & 0 & 0 \\ L_n M & 0 & 0 & -\mu_n(1-\check{\Delta}_n) & 0 & 0 & 0 \\ J_1 M & 0 & 0 & 0 & -\nu_1(1-\check{\delta}_1) & 0 & 0 \\ \vdots & 0 & 0 & 0 & 0 & \ddots & 0 \\ J_m M & 0 & 0 & 0 & 0 & 0 & -\nu_m(1-\check{\delta}_m) \end{bmatrix} < 0,$$

where  $N := \gamma^2 + (2L_0 - \sigma)M + M^2 + \sum_{i=1}^n \mu_i + \sum_{j=1}^m \nu_j \bar{\delta}_j^2$ ,

hold, then the set  $\mathcal{W} := \{(x, e, \tau_1, \kappa, \tau_2) : x = \mathbf{0}_{n_x}, e = \mathbf{0}_{n_e}\}$  is UGpAS for (10)-(11).



# Example

- plant with delayed input:

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t - \delta), \quad (26a)$$

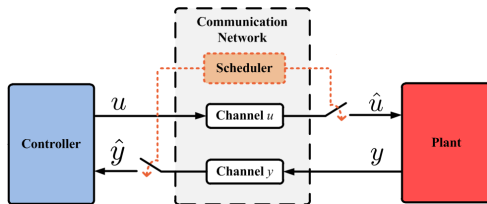
$$y(t) = C_p x_p(t), \quad (26b)$$

- dynamic output feedback observer-predictor controller:

$$\dot{x}_{c1}(t) = A_c x_{c1}(t) + B_c \left[ e^{A_p \delta} x_{c2}(t) + \int_{t-\delta}^t e^{A_p(t-\theta)} B_p u(\theta) d\theta \right], \quad (27a)$$

$$\dot{x}_{c2}(t) = A_p x_{c2}(t) + B_p u(t - \delta) + L [y(t) - C_p x_{c2}(t)], \quad (27b)$$

$$u(t) = C_c x_{c1}(t), \quad (27c)$$

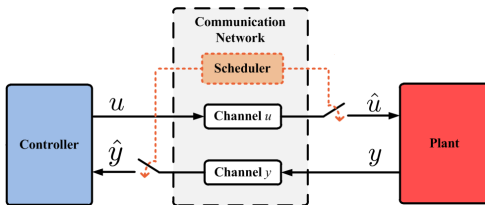


# Communication Network

- error vector:

$$e(t) = \begin{bmatrix} e_y(t) \\ e_u(t) \end{bmatrix} = \begin{bmatrix} \hat{y}(t) - \int_{t-\delta}^t y(s) ds \\ \hat{u}(t) - u(t) \end{bmatrix}$$

and  $\ell = 2$

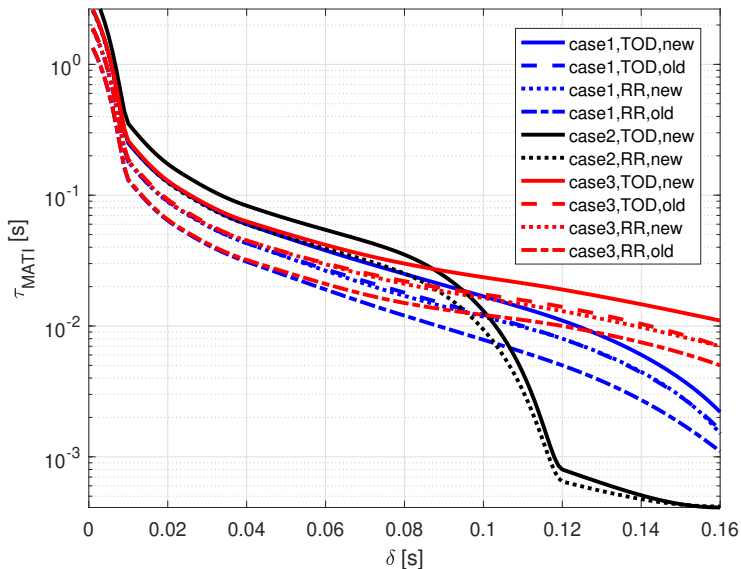


# Three Cases

- **Case 1:**  $u$  and  $y$  in (26a) and (27b), respectively, are replaced with  $\hat{u}$  and  $\hat{y}$ , respectively,
- **Case 2:** Case 1 together with  $\hat{u}$  instead of  $u$  in (27a)-(27b), and
- **Case 3:** Case 2 augmented with the model-based predictor  $\dot{\hat{u}} = C_c B_c \int_{t-\delta}^t e^{A_p(t-\theta)} B_p \hat{u}(\theta) d\theta$ .<sup>[7]</sup>

<sup>[7]</sup> Contribution 2 & 4

# Numerical Results



# Future Work

- more generic Lypunov-Krasovskii functionals
- optimal parameters in the presented stability conditions ✓ [8]
- extend MATIs using Minimal Allowable Transmission Intervals (MIATIs)<sup>[9]</sup>
- event-<sup>[10]</sup> and self-triggering
- account for noise and disturbances<sup>[11]</sup>

[8] Domagoj Tolić. “Stabilizing transmission intervals and delays in nonlinear networked control systems through hybrid-system-with-memory modeling and Lyapunov–Krasovskii arguments”. In: *Nonlinear Analysis: Hybrid Systems* 36 (2020), p. 100834.

[9] S.H.J. Heijmans et al. “Computing Minimal and Maximal Allowable Transmission Intervals for Networked Control-Systems Using the Hybrid Systems Approach”. In: *IEEE Control Systems Letters* 1.1 (2017), pp. 56–61.

[10] V. S. Dolk, D. P. Borgers, and W. P. M. H. Heemels. “Output-Based and Decentralized Dynamic Event-Triggered Control With Guaranteed Lp-Gain Performance and Zeno-Freeness”. In: *IEEE Trans. on Automatic Control* 62.1 (2017), pp. 34–49.

[11] Domagoj Tolić and Sandra Hirche. “Stabilizing Transmission Intervals for Nonlinear Delayed Networked Control Systems”. In: *IEEE Trans. on Automatic Control* 62.1 (2017), pp. 488–494.

# Thanks

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**Questions? Comments?  
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**Thank You for Your attention!!**