3.10pt

Frequency isolation problem for hyperbolic QEP

Suzana Miodragović

UNIVERSITY J. J. STROSSMAYER OF OSIJEK
DEPARTMENT OF MATHEMATICS
Trg Ljudevita Gaja 6
31000 Osijek, Croatia
http://www.mathos.unios.hr

ssusic@mathos.hr

Joint work with:

J. Moro, F. de Teran, N. Truhar

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Summary

- Hyperbolic quadratic eigenvalue problem
- Frequency isolation algorithms
  - Basic isolation algorithm
  - Continuation algorithm
- Numerical examples

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Motivated by: J.Moro and J.Egaña, *Directional algorithms for the frequency isolation problem in undamped vibrational systems*, Mechanical Systems and Signal Processing, 2016.

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Problem

When the eigenvalues of the QEP are in certain region, vibration system experiences dangerous vibrations (resonance) and M, D and K should be chosen in such way that this spectral regions are avoided.

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More precise - we fix a certain tolerance ρ and define a so-called resonance band

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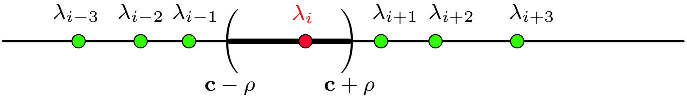

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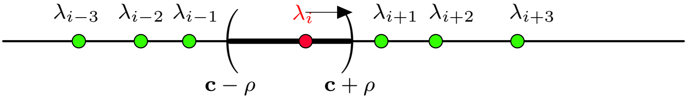

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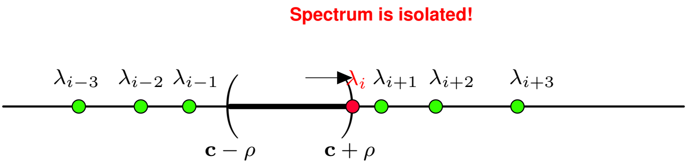

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Given resonance band $\mathcal{R} = (c - \rho, c + \rho)$ and vibrational system (M, D, K) with some eigenvalue in $(c - \rho, c + \rho)$, modify system in such way that the new system $(M + \Delta M, D + \Delta D, K + \Delta K)$

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- has no eigenvalue in the resonance band and
- is close (in some sense) to original system (M, D, K)

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Theorem (F.Tisseur, N.J.Higham, P. Van Doren '02)

A system $\lambda^2 M + \lambda D + K$ with M Hermitian positive definite and D and K Hermitian is hyperbolic if the following inequality holds:

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Corollary

Let $\lambda^2 M + \lambda D + K$ be hyperbolic and ΔD a Hermitian perturbation of the damping matrix, D, such that

$$\| \Delta D \|_2 < \sigma_{\min}(D) - 2\sqrt{\lambda_{\max}(M)\lambda_{\max}(K)}\,.$$

Then the perturbed system $\lambda^2 M + \lambda(D + \Delta D) + K$ is hyperbolic.

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Work in parametar space $\mathbb{R}^{2n-1}$ instead in matrix space $\mathbb{R}^{n \times n}$!

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2. **Isolation:** Given w_{max} from **Stage 1.**, find smallest $\alpha^* \in \mathbb{R}$ such that eigenvalues correspond to $s = s_0 + \alpha^* w_{max}$ are outside the $\mathcal{R}$.

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- Orthonormal basis of $W^{\perp}$ (e.g. via QR factorization)
- w_{max} is singular vector that correspond to σ_{max} of scalar product matrix, that is $\Pi \in \mathbb{R}^{q \times q}$ with

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OVERALL COST: $O(n^3)$

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That is α is between the quantities:

$$\tau^- = \sqrt{\lambda_{\max}(M)\lambda_{\max}(K)} - \sigma_{\min}(D) \quad \text{and} \quad \tau^+ = \sigma_{\min}(D) - \sqrt{\lambda_{\max}(M)\lambda_{\max}(K)}\,.$$

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are inside the resonance band $\mathcal{R}$. As soon as the number of eigenvalues in $\mathcal{R}$ is zero - STOP.

OVERALL COST: $O(n)$ per bisection step

Quadratic eigenvalue problem have to stay **hyperbolic**!
Algorithm works only if there are no eigenvalues in $\mathcal{R}$ either for $\alpha = \tau^-$ or $\alpha = \tau^+ \longrightarrow$ provides starting interval for bisection.

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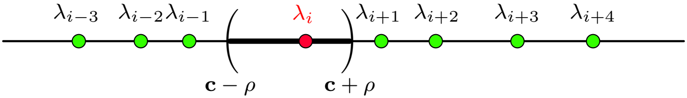

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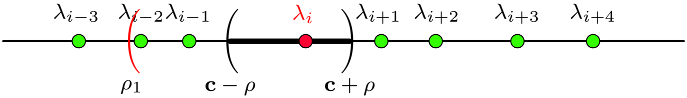

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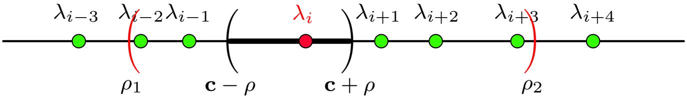

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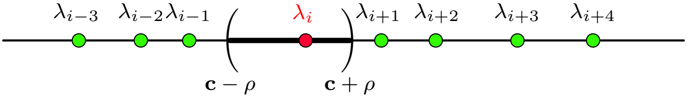

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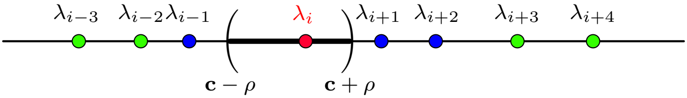

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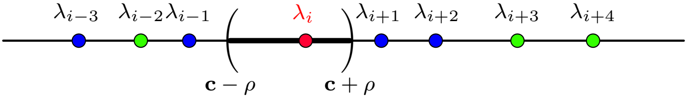

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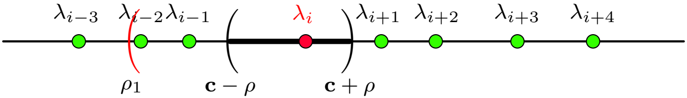

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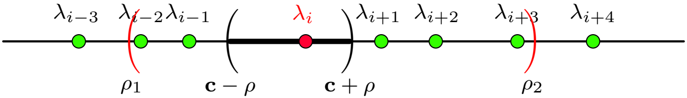

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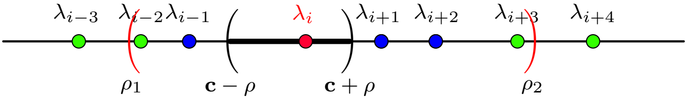

Numerical example

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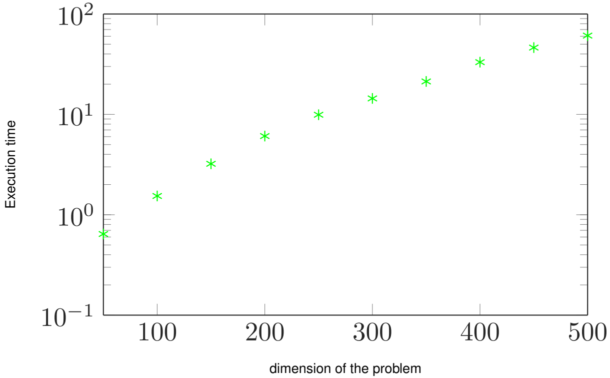

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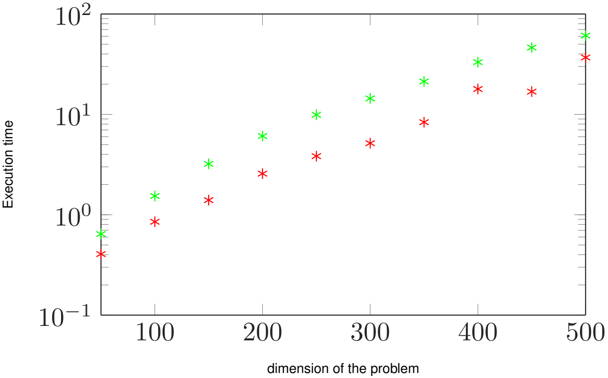

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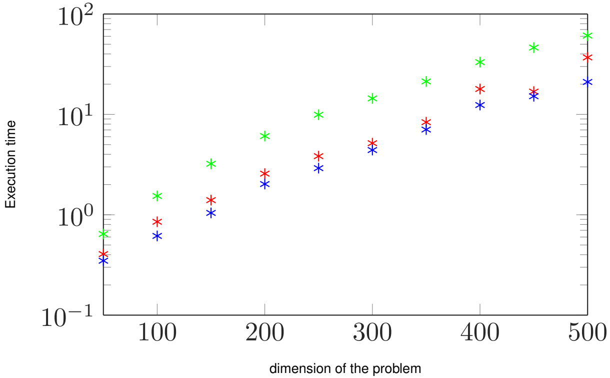

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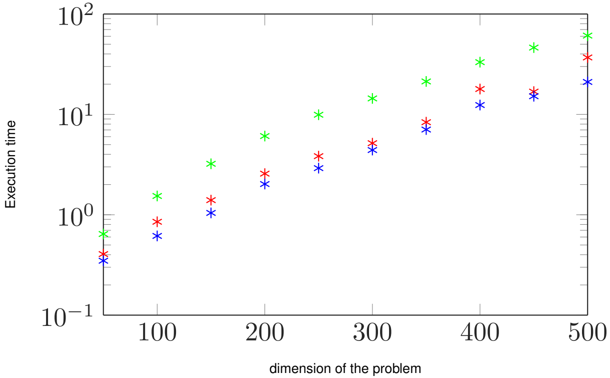

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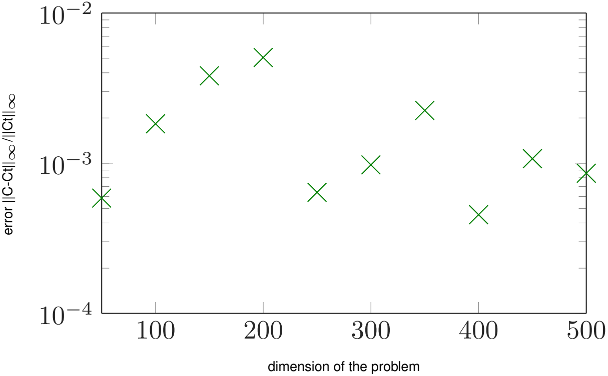

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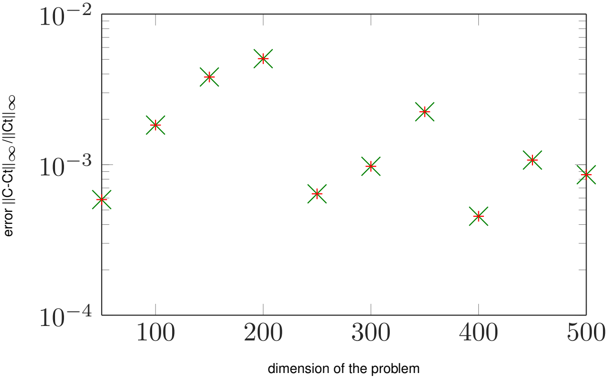

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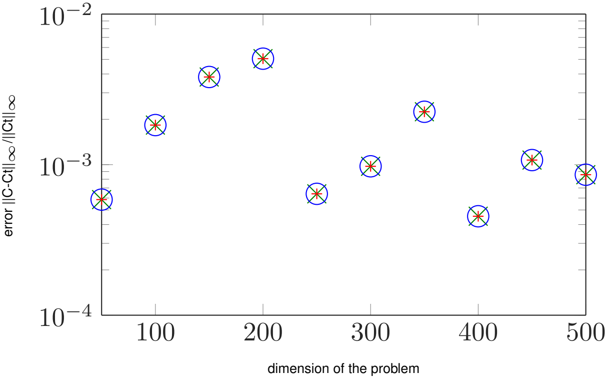

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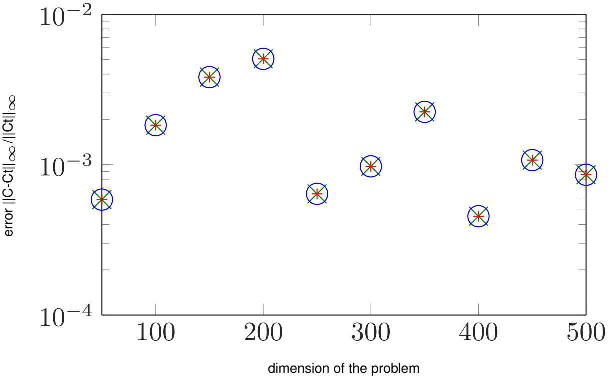

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n	I_{out}	I_{out}^1	I_{out}^2	$Time$	$Time_1$	$Time_2$	$Error$
50	6	6	1	0.79	0.36	0.31	$5.8582e-04$
100	14	14	13	1.98	0.84	0.55	$1.8381e-03$
150	8	8	8	2.91	1.47	1.00	$3.8210e-03$
200	17	17	14	5.64	2.50	1.96	$5.0559e-03$
250	7	7	4	9.37	3.78	2.88	$6.3999e-04$
300	5	5	5	13.51	4.96	4.37	$9.7486e-04$
350	6	6	6	20.14	8.46	6.97	$2.2452e-03$
400	7	7	7	31.72	16.54	11.25	$4.4521e-04$
450	6	6	6	45.94	15.81	14.12	$1.0724e-03$
500	17	17	17	58.93	26.64	19.56	$8.5720e-04$

Table: Set I_{out} before and after selection of "dangerous" eigenvalues for intervals $(0, 18)$, $(c - \rho - 3, c + \rho + 3)$ and $(c - \rho - 0.3, c + \rho + 0.3)$.

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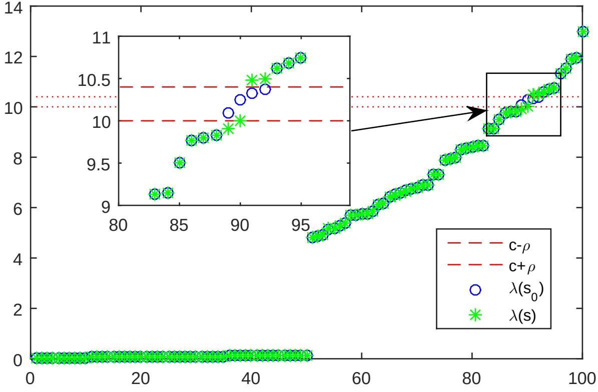
- If not, take h_i take $h_i = \tau_i^+$ or $h_i = \tau_i^-$ and continue.

Example-continuation algorithm

M, D, K tridiagonal s.t. QEP hyperbolic, $n = 50$
Isolation of eigenvalues $\lambda_{89}(s_0) = 10.0952$, $\lambda_{90}(s_0) = 10.2558$, $\lambda_{91}(s_0) = 10.3211$, $\lambda_{92}(s_0) = 10.3778$ from the resonance band $(c - \rho, c + \rho) = (10, 10.4)$.



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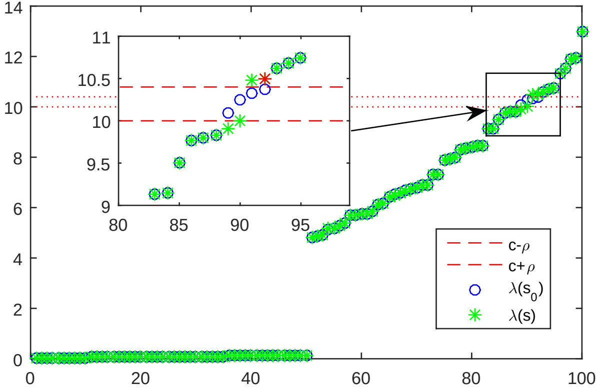

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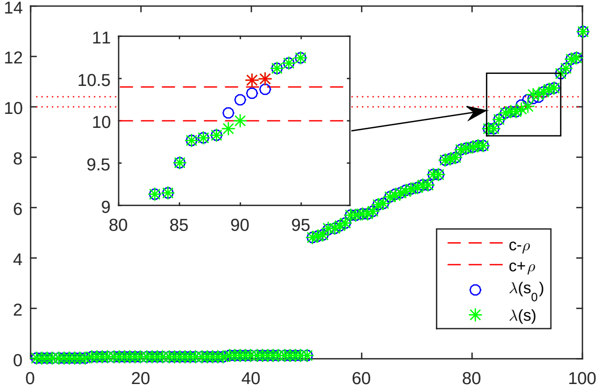

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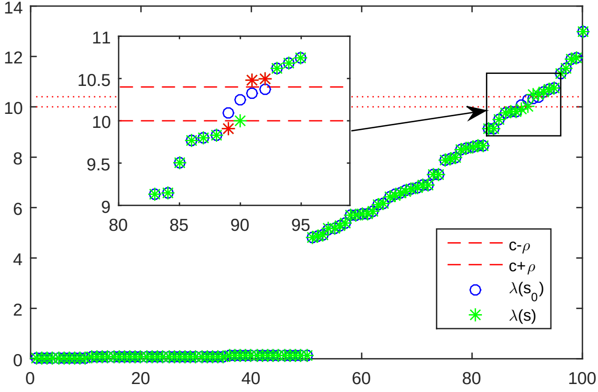

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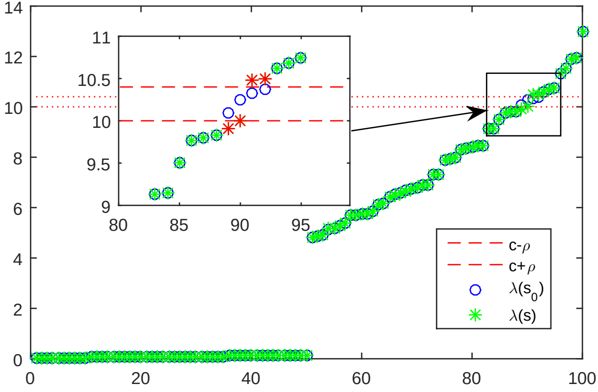

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M, K and D are chosen such that the system is stable $\rightarrow$ all eigenvalues are purely imaginary and semi-simple.

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Numerical example

$*$ are einegvalues with indices in set I_{out} for different tolerance Tol_1

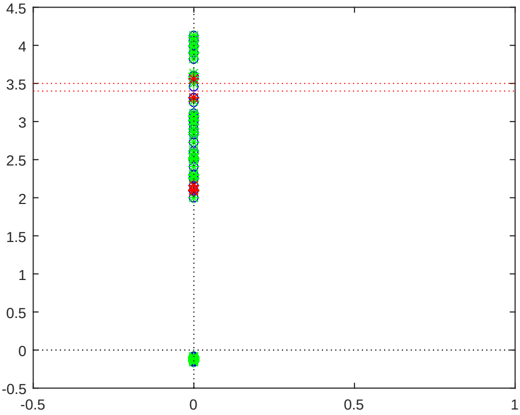

Figure: $Tol_1 = 0.5$

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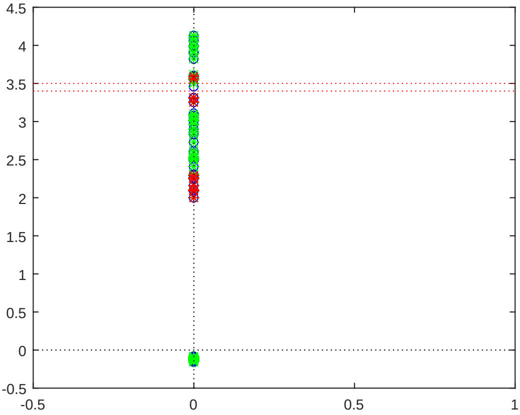

Figure: $Tol_1 = 1$

Numerical example

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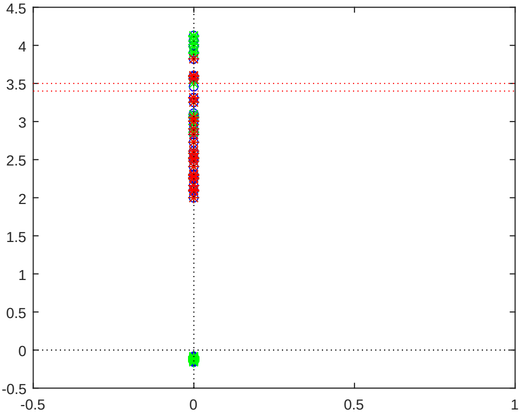

Figure: $Tol_1 = 2$

Numerical example

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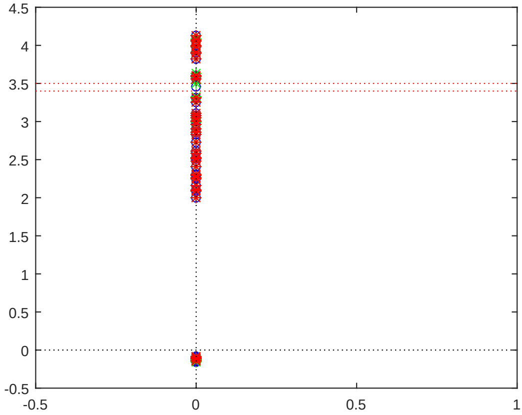

Figure: $Tol_1 = 4$

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Thank you for attention!