Market-based power systems

A control perspective

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smart grids ?

smart grids ?

hidden technology

smart grids ?

hidden technology

invisible hand

smart grids ?

hidden technology

invisible hand of market

Acknowledgments & Background

A.J., PhD thesis “Price-based optimal control of electrical power systems”, TU Eindhoven, 2007.

Paul van den Bosch, Mircea Lazar, Ana Virag, Ralph Hermans, Siep Weiland, Jasper Frunt, Frank Nobel
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(FSB, Zagreb)

Outline

1. Power system basics
2. Market-based operation (STATIC)
3. Congestion management (STATIC)
4. Market-based robust spatial distribution of ancillary services (STATIC + DYNAMIC)
5. Closed loop price-based operation (DYNAMIC)
   - Optimization as feedback controller
   - Price-based control of power systems
6. Conclusions

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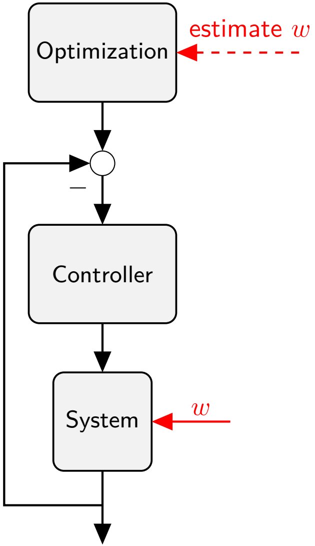
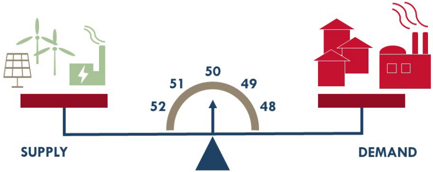
Power system operation



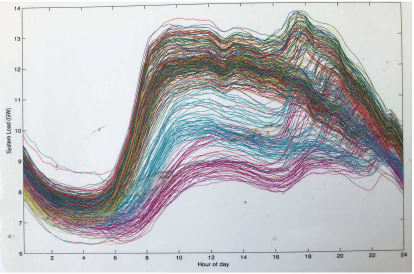
Offline optimization: dispatch based on forecasts of renewables and loads

Re-scheduling set-points: to mitigate severe forecasting errors (redipatch)

Online control: based on frequency



Challenge I: Uncertainties (renewable energy sources)



Traditional

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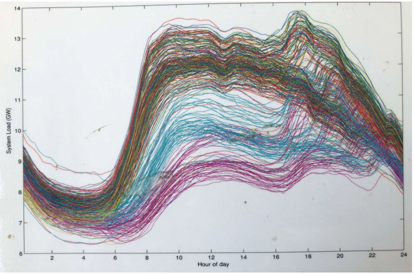

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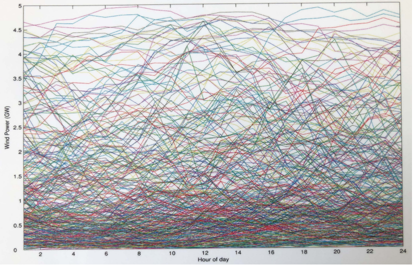

Renewables

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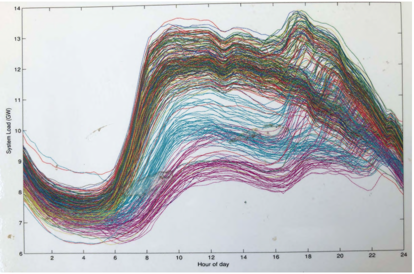

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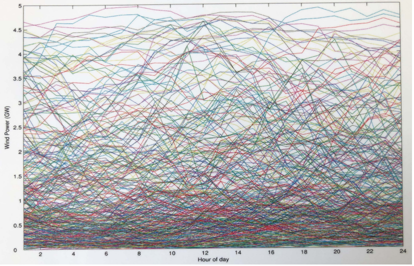


Renewables

- Traditional operation to a big extent relies on repetitiveness
- increased uncertainties & fluctuations → inefficient and infeasible to separate optimization and control

Challenge I: Uncertainties (renewable energy sources)

increased uncertainties & fluctuations → inefficient and infeasible to separate optimization and control

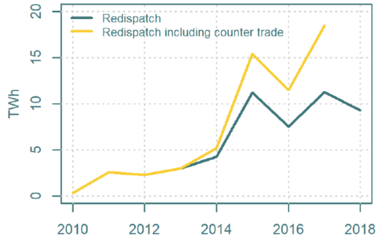
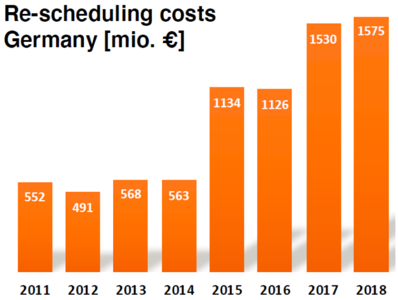


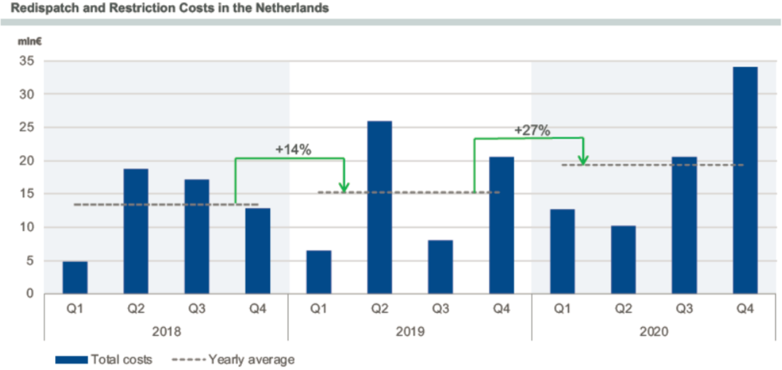
Figure 2.1.: *Development of Redispatch Volumes in TWh in Germany. Sources: Bundesnetzagentur and BDEW.*



[Bundesnetzagentur, Monitoringbericht 2011-2019]

Congestion Management Costs NL

Costs for Congestion Management increased by 27% in 2020



- This figure shows redispatch and restriction costs in the Netherlands. Restriction concerns contracts with market parties to withhold a share of production for a certain period. Total costs increased from €61,0 million in 2019 to € 77,6 million in 2020 with a slight increase of redispatch volume activated. A significant part of the cost increase is related to restriction contracts.

Challenge II: Deregulation

electrical energy is a peculiar product: no large scale buffering, no routing, trades in energy (economy) and provision in power (physics/control) - no straightforward transfer of knowledge from other fields

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Optimization: Maximizing social welfare

Feedforward, slow (energy time-scale)

Primal

$$\min_{\underbrace{\{p_i \in \mathcal{P}_i\}}_{\text{PRODUCTION}},\underbrace{\{d_j \in \mathcal{D}_j\}}_{\text{DEMAND}}} \overbrace{\sum_{i=1}^{n} C_i(p_i)}^{\text{COSTS}} - \overbrace{\sum_{j=1}^{m} B_j(d_j)}^{\text{BENEFITS}} \qquad (= \text{max social welfare})$$

$$\text{subject to} \quad \sum_{i=1}^{n} p_i = \sum_{j=1}^{m} d_j \qquad (\text{balance supply and demand})$$

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Dual

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda)$$

where

$$\ell(\lambda) = \min_{p_i \in \mathcal{P}_i, d_j \in \mathcal{D}_j} \quad \sum_{i=1}^{n} C_i(p_i) - \sum_{j=1}^{m} B_j(d_j) + \lambda \left(\sum_{j=1}^{m} d_j - \sum_{i=1}^{n} p_i \right)$$

Assumption: convexity. $C_i(\cdot)$ convex functions, $B_j(\cdot)$ concave fun., $\mathcal{P}_i, \mathcal{D}_j$ convex sets.

Optimization: Maximizing social welfare - Energy market

Market operator

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda) \quad \Leftrightarrow \quad \text{determine } \lambda : \sum_{j=1}^{m} d_j^\star = \sum_{i=1}^{n} p_i^\star$$

Rational behaviour of market players (max its own benefits)

Supplier's *local* minimizations

$$p_1^\star = \text{argmin}_{p_1 \in \mathcal{P}_1} \quad C_1(p_1) - \lambda p_1$$
$$p_2^\star = \text{argmin}_{p_2 \in \mathcal{P}_2} \quad C_2(p_2) - \lambda p_2$$
$$\vdots$$
$$p_n^\star = \text{argmin}_{p_n \in \mathcal{P}_n} \quad C_n(p_n) - \lambda p_n$$

Demand's *local* minimizations

$$d_1^\star = \text{argmin}_{d_1 \in \mathcal{D}_1} \quad \lambda d_1 - B_1(d_1)$$
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λ^* **which solves the above problem is the (market clearing) price**

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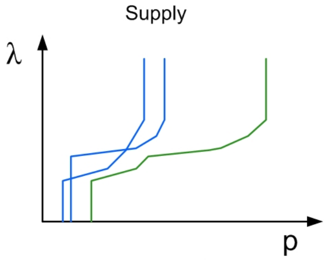

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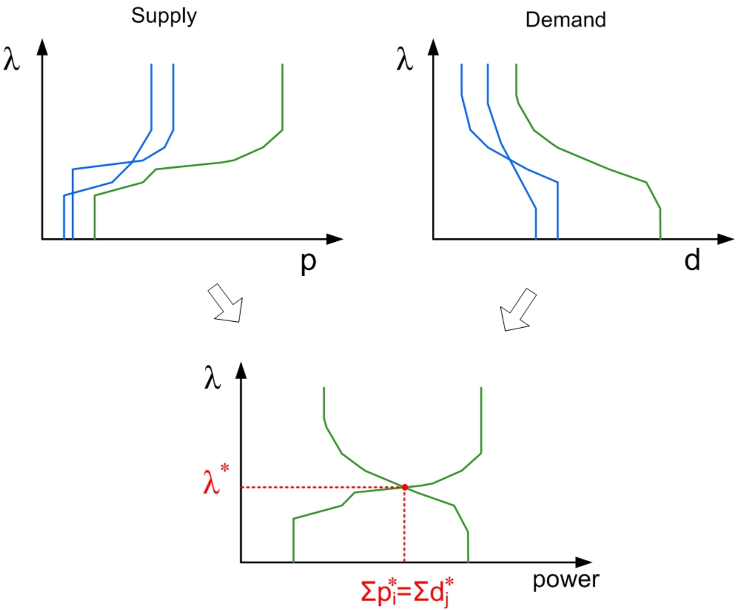
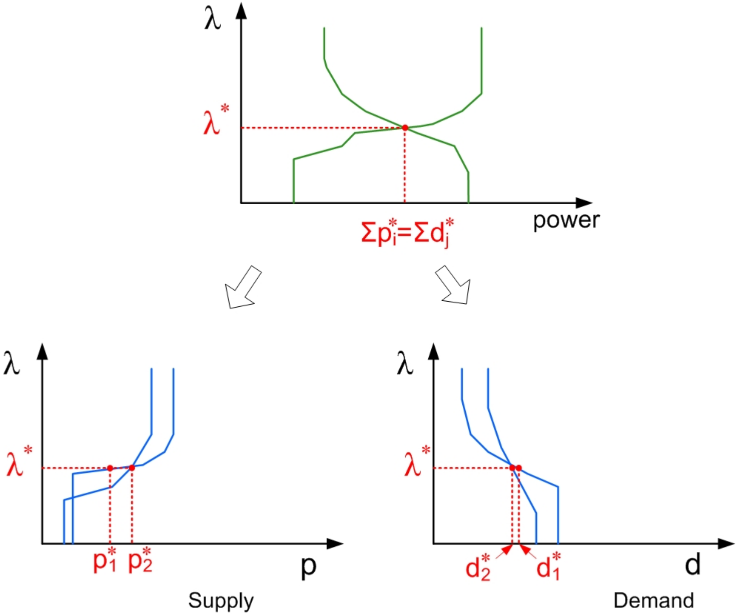


blue curves - BIDS: $\beta_i(p_i)$, $\beta_i(d_i)$ $\rightarrow$ incremental costs (in perfect competition)



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Benefits of market-based (price-based) operation

In mathematical terms we reached (via dual) the same solution (as primal).
Why deregulation? new solution architecture: new players (market operators, competing parties),definition of who does what, prices and bids as protocols for coordination

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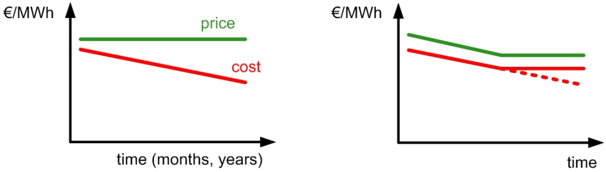

a) price-cap regulation

b) cost of service regulation

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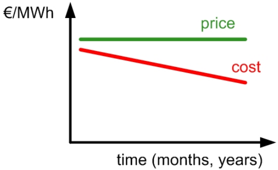
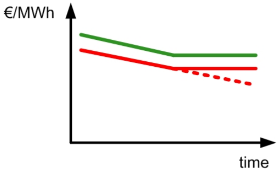
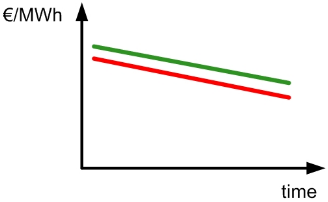


a) price-cap regulation



b) cost of service regulation



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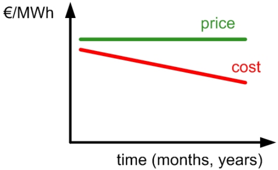

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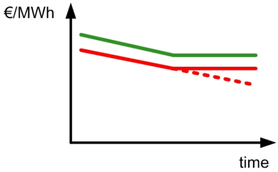

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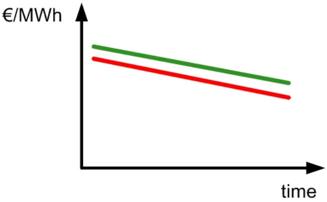

"invisible hand of market" (*Solution architecture matters*)

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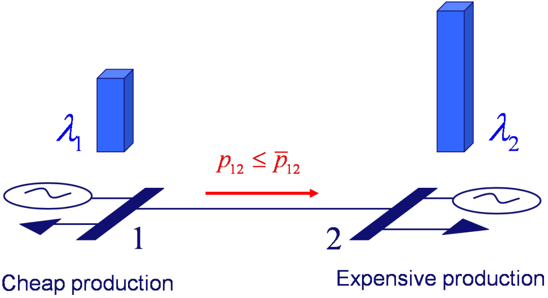
Congestion management

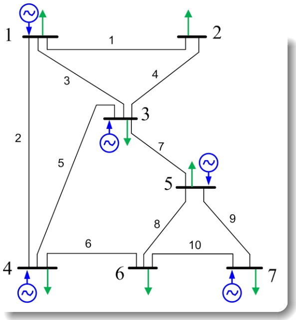


Line flow limits:

- physical: thermal limits, stability limits
- contingency limits (robustness): physical limits following contingency

Congestion is a problem on more time-scales (day-ahead, real-time).

Power flow equations (DC approximation)



$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} = \begin{pmatrix} b_{\mathcal{N}_1} & -b_{12} & \dots & -b_{1n} \\ -b_{12} & b_{\mathcal{N}_2} & \dots & -b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -b_{1n} & -b_{2n} & \dots & b_{\mathcal{N}_n} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix}$$

with $b_{\mathcal{N}_i} := \sum_{j \in \mathcal{N}_i} b_{ij}$

Power flow equations

$$p = B\theta$$

Remark: $B^\top = B, \quad B\mathbf{1}_n = 0.$

Line flow limits

$$L\theta \leq \overline{e}_{\mathcal{E}}$$

Nodal power injections:

$p_i < 0$ consumption,
$p_i > 0$ production

DC line power flow model:

$p_{ij} = b_{ij}(\theta_i - \theta_j) = -p_{ji}$
b_{ij} = susceptance of line
θ_i = voltage phase angle

Optimal power flow problem

$p_i =$ node aggregated controllable power injection with assigned economic objective function $J_i(p_i)$:

- $p_i < 0$, net consumption, $J_i(p_i) = -B_i(p_i)$
- $p_i > 0$, net production, $J_i(p_i) = C_i(p_i)$

$q_i =$ uncontrollable, price inelastic, nodal power injection (net consumption: $q_i < 0$, net production : $q_i > 0$).

Optimal power flow problem (OPF)

$$\begin{aligned} \min_{p,\theta} \quad & \sum_{i=1}^{n} J_i(p_i) \\ \text{subject to} \quad & p + q - B\theta = 0 \\ & \underline{p} \leq p \leq \overline{p} \\ & L\theta \leq \overline{e}_{\mathcal{E}} \end{aligned}$$

Nodal pricing

KKT conditions (after "including back" the limits $\{\underline{p}_i, \overline{p}_i\}$ into the bids $\beta_i(p_i)$)

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$$\text{subject to } p - B\theta = 0$$
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$$L\theta \leq \overline{e}_{\mathcal{E}}$$

KKT conditions

$$\beta(p^\star) - \lambda^\star = 0$$
$$p^\star - B\theta^\star = 0$$
$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \leq (-L\theta^\star + \overline{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star \geq 0$$

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Singe price in case of no congestion

$$-L\theta^\star + \overline{e}_{\mathcal{E}} < 0 \quad \Longrightarrow \quad \mu^\star = 0 \quad \Longrightarrow \quad B\lambda^\star = 0 \quad \Longrightarrow \quad \lambda^\star = \mathbf{1}_n \hat{\lambda},\ \hat{\lambda} \in \mathbb{R}$$

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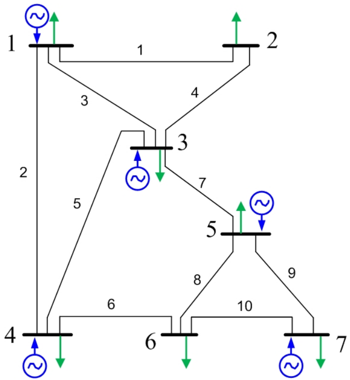
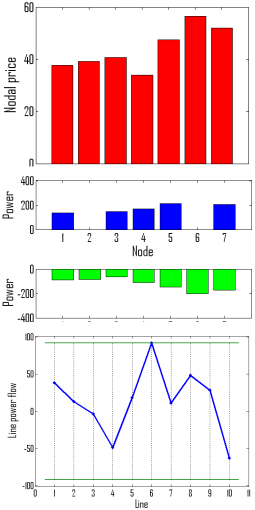
In case of singe congested line, optimal nodal price in general have different value for each node. $(B\lambda^\star = -L^\top \mu^\star)$

Nodal pricing

Example





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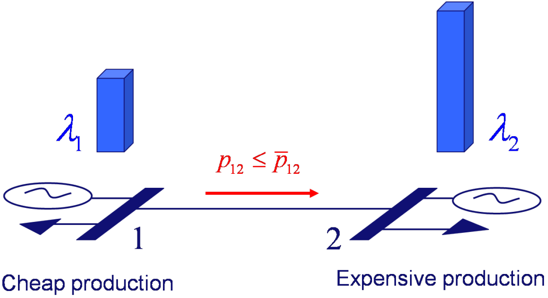
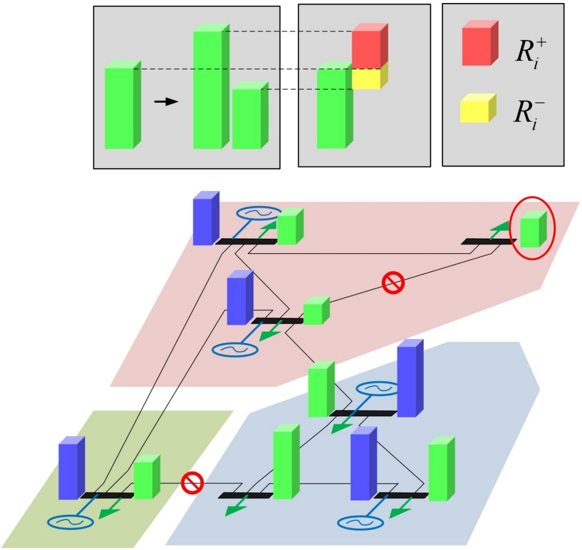
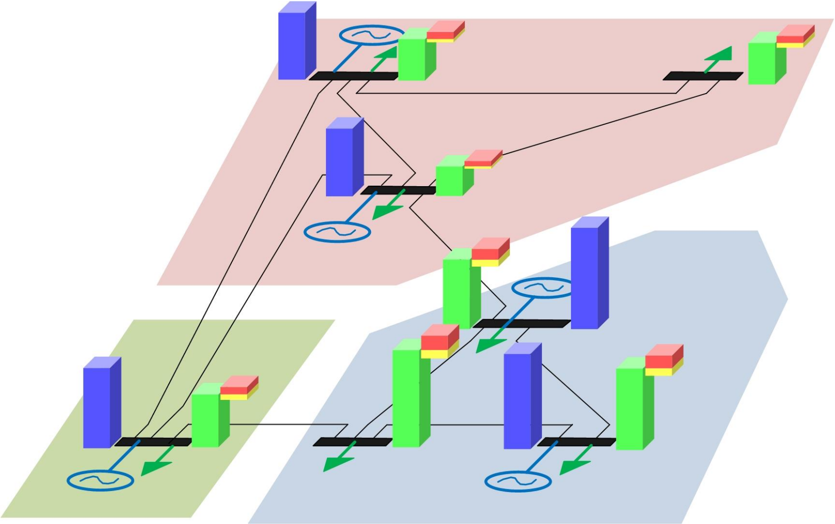
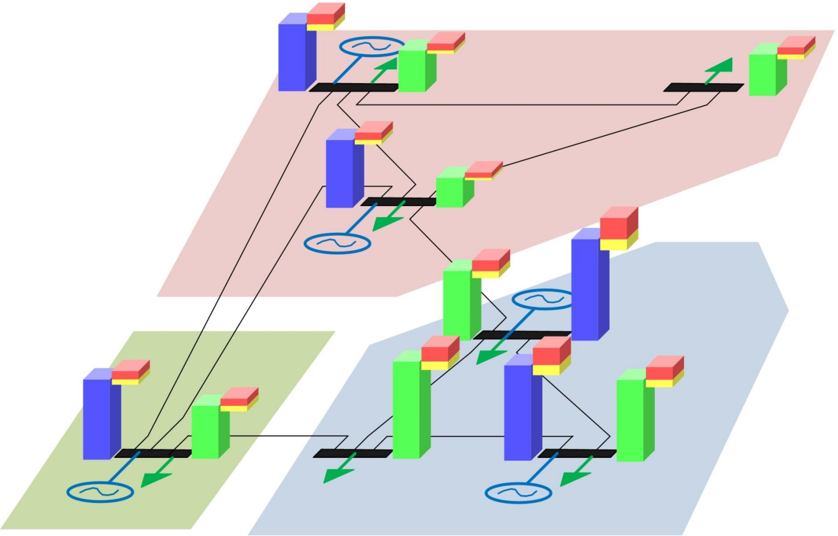
Spatial allocation of reserves (balancing ancillary services)

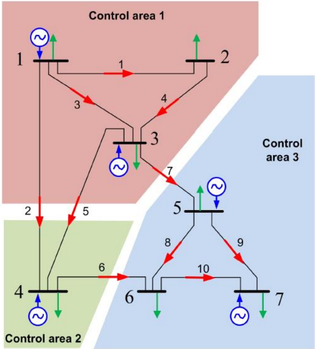


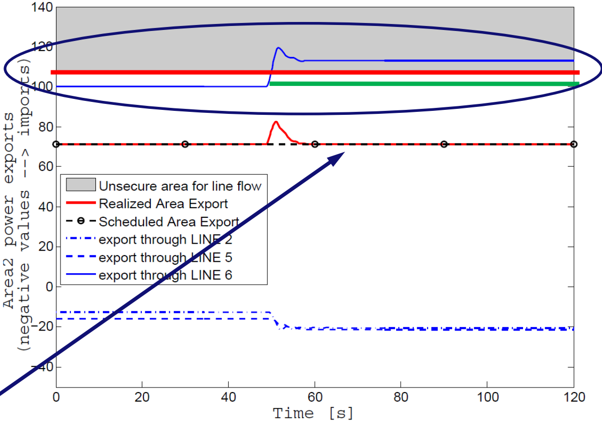
Allocation of cheaper reserves (NODE 1) behind congested lines cannot cover for uncertain fluctuations in NODE 2.


R_i^+
R_i^-









In current system, reliability is accounted for in "aggregated" form here

RELIABILITY MARGIN

Size of reliability margin: reliability vs. efficiency trade-off

Economically optimal working point is often on the border of feasible region

Problem definition

Robust congestion constraints

The participation function - a priory fixed structure (more options make sense), parameters are gains in secondary control loops

$$f(t) = \gamma(\tilde{a}^+(k), \tilde{a}^-(k), q(t))$$

$\tilde{a}^+(k) =$ purchased and allocated up-regulating AS
$\tilde{a}^-(k) =$ purchased and allocated down-regulating AS
$\tilde{a}^+(k)$ and $\tilde{a}^-(k)$ are **vectors defining spatial distribution of AS**

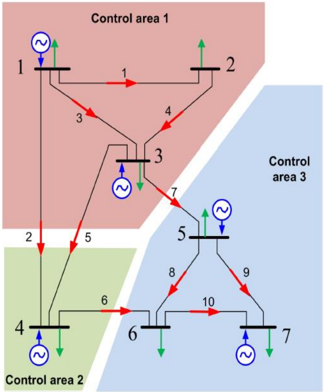
Uncertainty model

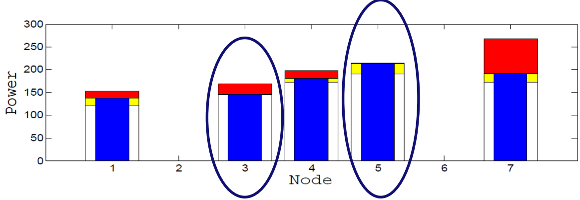
$$q(t) \in \tilde{\mathcal{Q}}(k) = \{\, q \mid q = \tilde{R}(k)w,\, w \in \tilde{\mathcal{W}}(k) \subset \mathbb{R}^m \}$$
$$\tilde{\mathcal{W}}(k) = \mathrm{conv}\{\tilde{w}_1(k), \ldots, \tilde{w}_T(k)\}, \qquad 0 \in \tilde{\mathcal{W}}(k)$$

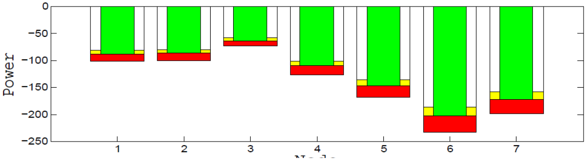
Robust congestion constraints

$$L\delta \leq \Delta\tilde{l}(k) \qquad \text{for all} \quad \delta \in \tilde{\mathcal{D}}(k) \text{ where}$$

$$\tilde{\mathcal{D}}(k) := \{\delta \mid \begin{array}{l} \tilde{R}(k)w + \gamma\left(\tilde{a}^+(k), \tilde{a}^-(k), \tilde{R}(k)w\right) = B\delta, \\ w \in \tilde{\mathcal{W}}(k) \end{array} \}$$



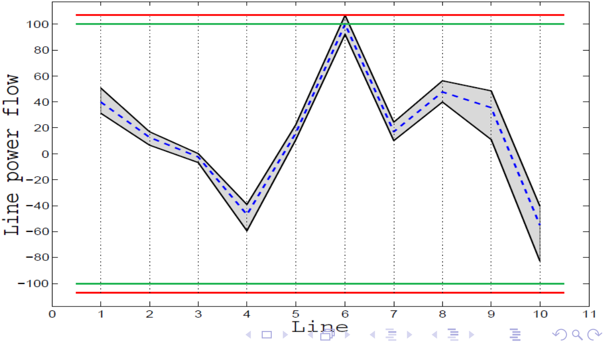


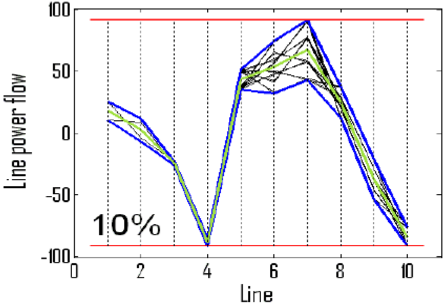


Spatial distribution of AS:
Shaping the “uncertainty tube”→

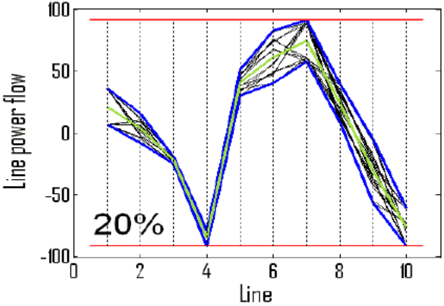
Get reliability for best costs

Possible to include optimal cooperation between control areas

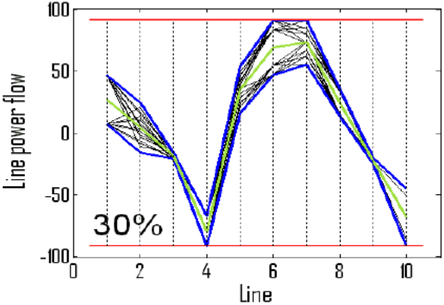




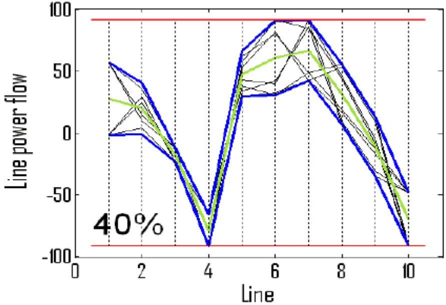
(a) Power flows for 10% uncertainty level.



(b) Power flows for 20% uncertainty level.

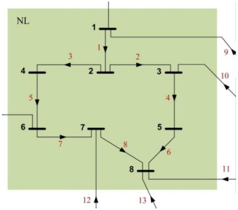


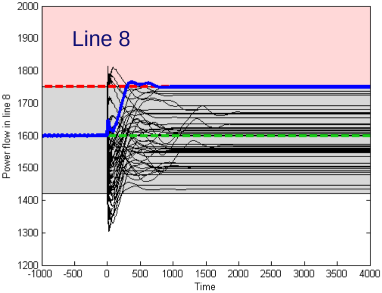
(c) Power flows for 30% uncertainty level.

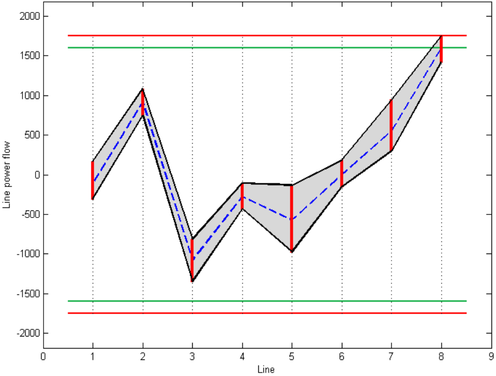


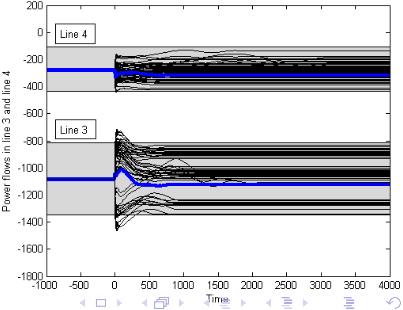
(d) Power flows for 40% uncertainty level.

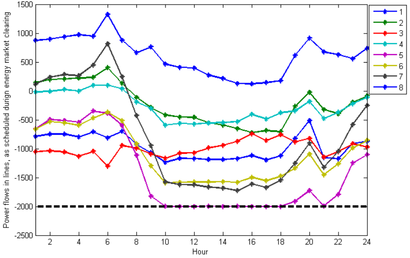
Optimized uncertainty in line power flows

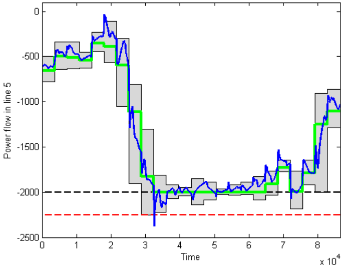


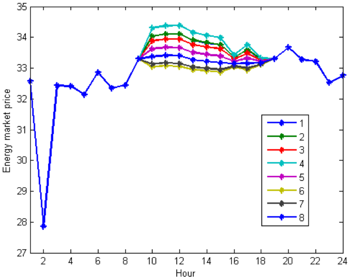


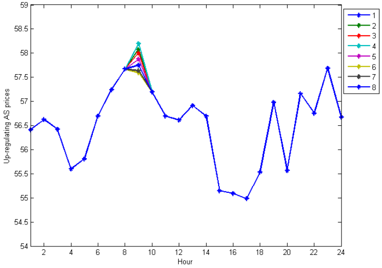





Power flows in lines, as scheduled durign energy market clearing
Hour
1
2
3
4
5
6
7
8


Power flow in line 5
Time
x 10^4

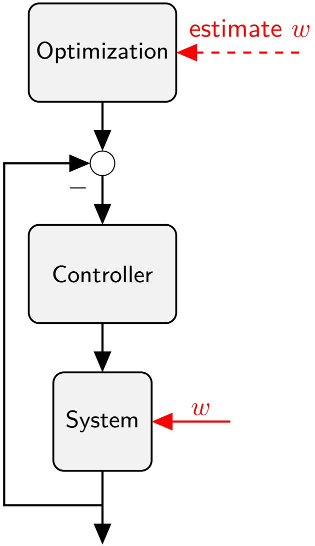

Energy market price
Hour
1
2
3
4
5
6
7
8


Up-regulating AS prices
Hour
1
2
3
4
5
6
7
8

Outline

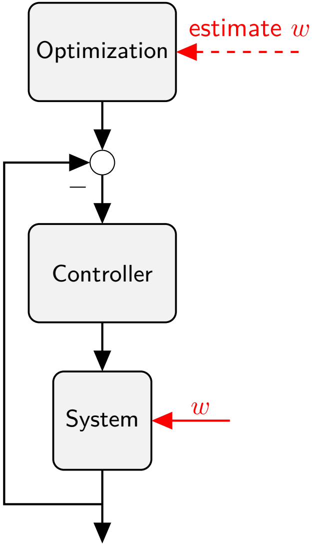
1. Power system basics
2. Market-based operation (STATIC)
3. Congestion management (STATIC)
4. Market-based robust spatial distribution of ancillary services (STATIC + DYNAMIC)
5. Closed loop price-based operation (DYNAMIC)
   - Optimization as feedback controller
   - Price-based control of power systems
6. Conclusions

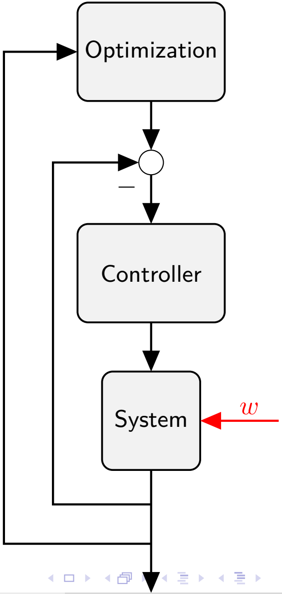
Optimization as feedback controller



Increased uncertainties & fluctuations $\rightarrow$ inefficient and infeasible to separate optimization and control

Optimization as feedback controller





Theory literature inspired by power systems

survey slide by Florian Dorfler, ETH Zurich

- lots of recent theory development stimulated by **power systems** problems

  [Simpson-Porco et al., 2013], [Bolognani et al, 2015], [Dall'Anese & Simmonetto, 2016], [Hauswirth et al., 2016], [Gan & Low, 2016], [Tang & Low, 2017], . . .

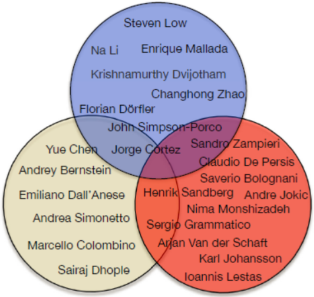
  > A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems
  >
  > Daniel K. Molzahn,* *Member, IEEE*, Florian Dörfler,† *Member, IEEE*, Henrik Sandberg,‡ *Member, IEEE*, Steven H. Low,§ *Fellow, IEEE*, Sambuddha Chakrabarti,¶ *Student Member, IEEE*, Ross Baldick,¶ *Fellow, IEEE*, and Javad Lavaei,** *Member, IEEE*

- **early adoption**: KKT control [Jokic et al, 2009]

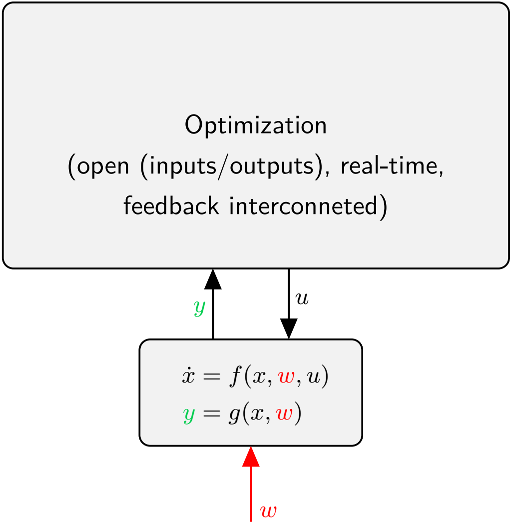
- literature **kick-started** $\sim$ 2013 by groups from Caltech, UCSB, UMN, Padova, KTH, & Groningen

- **changing focus**: distributed & simple $\rightarrow$ centralized & complex models/methods

- **implemented** in microgrids (NREL, DTU, EPFL, . . .) & conceptually also in transactive control pilots (PNNL)



Optimization in the loop (KKT controller)



Desired steady-state

$$\begin{aligned} \min_{y} \quad & J(y) \\ \text{subject to } \quad & Ly = h(w) \\ & g(y) \leq r(w) \\ & w \in \mathcal{W} \end{aligned}$$

-

Optimization in the loop (KKT controller)

$$
\begin{aligned}
\dot{x}_\lambda &= K_\lambda(\boxed{Ly - h(w)}) \\
\dot{x}_\mu &= L_\mu(\boxed{g(y) - r(w)}) + v \\
\dot{x}_c &= (L^\top x_\lambda + \nabla g(y) + \nabla J(y)) \\
0 \leq v \ \perp \ K_0 x_\mu &+ K_\mu(\boxed{g(y) - r(w)}) + v \geq 0 \\
u &= x_c
\end{aligned}
$$

y u

$$
\begin{aligned}
\dot{x} &= f(x, w, u) \\
y &= g(x, w)
\end{aligned}
$$

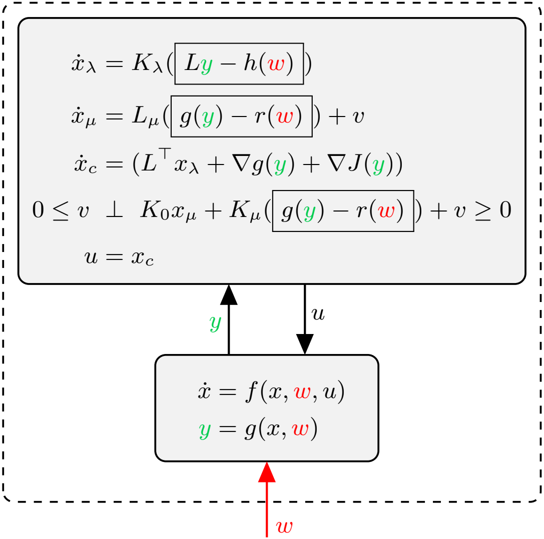
w

Desired steady-state

$$
\begin{aligned}
\min_{y} \quad & J(y) \\
\text{subject to } \; & Ly = h(w) \\
& g(y) \leq r(w) \\
& w \in \mathcal{W}
\end{aligned}
$$

-

Optimization in the loop (KKT controller)



Desired steady-state

$$\begin{aligned} \min_{y} \quad & J(y) \\ \text{subject to } & Ly = h(w) \\ & g(y) \leq r(w) \\ & w \in \mathcal{W} \end{aligned}$$

KKT conditions

$$\begin{aligned} \nabla J(y) + L^\top \lambda + \nabla g(y)\mu &= 0 \\ Ly - h(w) &= 0 \\ 0 \leq -g(y) + r(w) \ \perp \ \mu &\geq 0 \end{aligned}$$

buit into cl.-loop stead.stat.sys.

INTERMEZZO: complementarity systems

Complementarity system

$$\begin{aligned}\dot{x} &= f(x, w)\\ z &= h(x, w)\\ 0 \leq z &\perp w \geq 0\end{aligned}$$

$0 \leq z \perp w \geq 0$ is compact from for $z \geq 0,\ w \geq 0,\ z^\top w = 0$

- CS structure $\rightarrow$ rich theory (van der Schaft, Schumacher, Heemels, Camlibel, Brogliato, ...)
- Suitable framework for many applications (physics, optimization, economy), e.g.: constrained mech. systems, elect. circuits, optimal control, oligopolistic markets, Leontiev economy, ...

Toy example

System

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}-2.5 & 0 & -5\\ 0 & -5 & -15\\ 0.1 & 0.1 & -0.2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ -0.1\end{pmatrix} w + \begin{pmatrix}2.5 & 0\\ 0 & 5\\ 0 & 0\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}$$

$$\underbrace{\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}}_{y} = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

Optimal working point

$$\min_{y} \quad \begin{pmatrix}y_1\\ y_2\end{pmatrix}^{\top}\begin{pmatrix}3 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix} + \begin{pmatrix}-4 & -4\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$$

subject to $\boxed{y_1 + y_2 = w}$

$(y_1 - 4.7)^2 + (y_2 - 4)^2 \leq 3.5^2$

for $w \in [4, 11.5]$

Observe
$0.1y_1 + 0.1y_2 - 0.3y_3 = 0.1w$
in steady state

Toy example

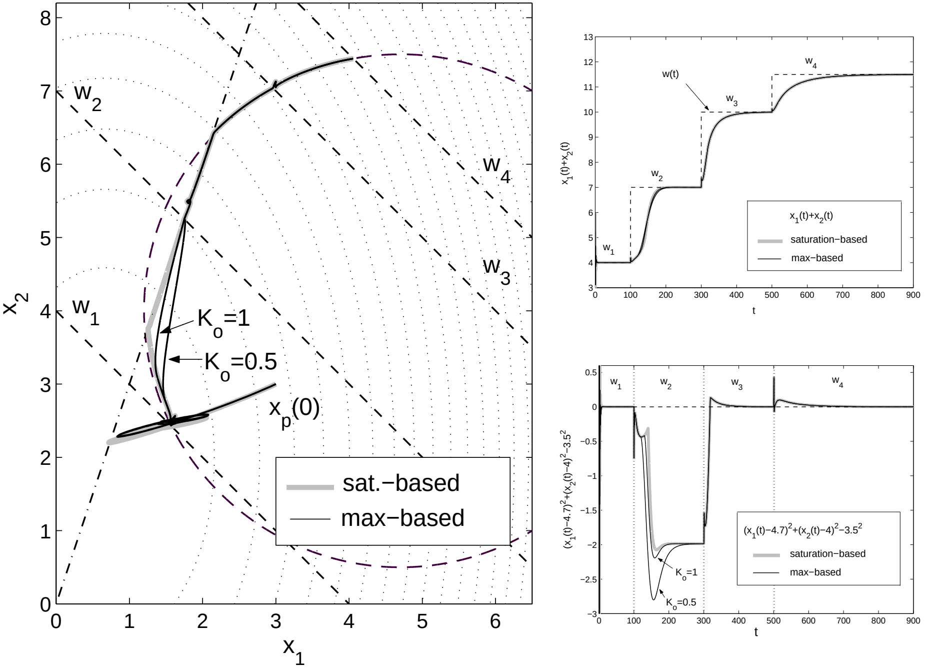
System

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}-2.5 & 0 & -5\\ 0 & -5 & -15\\ 0.1 & 0.1 & -0.2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ -0.1\end{pmatrix} w + \begin{pmatrix}2.5 & 0\\ 0 & 5\\ 0 & 0\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}$$

$$\underbrace{\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}}_{y} = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

The controller

$$\begin{aligned}
\dot{x}_\lambda &= K_\lambda\, x_3,\\
\dot{x}_\mu &= K_\mu((x_1-4.7)^2 + (x_2-4)^2 - 3.5^2 + v),\\
\dot{x}_c &= K_c\left(\begin{pmatrix}1\\ 1\end{pmatrix} x_\lambda + \begin{pmatrix}2x_1-9.4\\ 2x_2-8\end{pmatrix} x_\mu + \begin{pmatrix}6 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}-4\\ -4\end{pmatrix}\right),\\
0 \le v \perp (K_o x_\mu &+ (x_1-4.7)^2 + (x_2-4)^2 - 3.5^2 + v) \ge 0,\\
u &= x_c,
\end{aligned}$$


x_2
x_1
w_1
w_2
w_3
w_4
K_o=1
K_o=0.5
x_p(0)
sat.−based
max−based
x_1(t)+x_2(t)
w(t)
saturation−based
max−based
t
(x_1(t)−4.7)^2+(x_2(t)−4)^2−3.5^2
K_o=1
K_o=0.5

Distributed, real-time, price-based control of power sys.

$\Delta p_L = L\delta - \overline{e}_c$

Nodal pricing controller

$$\begin{pmatrix}\dot{x}_\lambda \\ \dot{x}_\mu\end{pmatrix} = \begin{pmatrix}-K_\lambda B & -K_\lambda L^\top \\ 0 & 0\end{pmatrix}\begin{pmatrix}x_\lambda \\ x_\mu\end{pmatrix} + \begin{pmatrix}-K_f & 0 \\ 0 & K_p\end{pmatrix}\begin{pmatrix}\Delta f \\ \Delta p_L + w\end{pmatrix},$$

$$0 \leq \; w \;\perp\; K_o x_\mu + \Delta p_L + w \;\geq\; 0,$$

$$\lambda = \begin{pmatrix}I_n & 0\end{pmatrix}\begin{pmatrix}x_\lambda \\ x_\mu\end{pmatrix},$$

$$\begin{aligned}
p - B\delta + \hat{p} &= 0, \quad \text{(global)}\\
B\lambda + L^\top\mu &= 0,\\
\nabla J(p) - \lambda + \nu^+ - \nu^- &= 0, \quad \text{(local)}\\
0 \leq \; (-L\delta + \overline{e}_c) \;\perp\; \mu &\geq 0,\\
0 \leq \; (-p + \overline{p}) \;\perp\; \nu^+ &\geq 0,\\
0 \leq \; (p + \underline{p}) \;\perp\; \nu^- &\geq 0
\end{aligned}$$

why does it work?

$$B\lambda + L^\top\mu + \Delta f^\star \mathbf{1} = 0,$$

$$\mathbf{1}^\top \begin{pmatrix}B & L^\top\end{pmatrix} = 0 \implies \mathbf{1} \notin \operatorname{Im}\begin{pmatrix}B & L^\top\end{pmatrix}$$

$$\implies \Delta f = 0,\; B\lambda + L^\top\mu = 0$$

Distributed, real-time, price-based control of power sys.

$$\Delta p_L = L\delta - \bar{e}_c$$

Nodal pricing controller

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$

$$0 \leq \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \geq \ 0,$$

$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

- no knowledge of cost/benefit functions of producers/consumers required
- required no knowledge of actual power injections (FEEDBACK!)
- required: B and L
- preserves the structure of B and L

Distributed, real-time, price-based control of power sys.

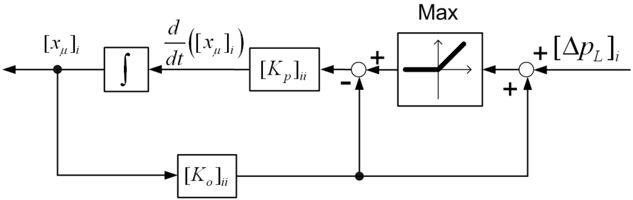
$\Delta p_L = L\delta - \overline{e}_c$

Nodal pricing controller

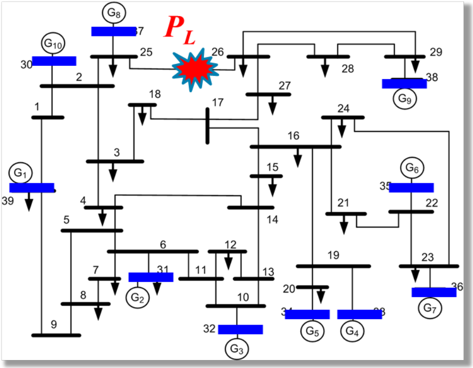
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$

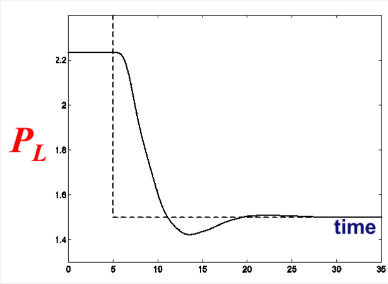
$$0 \leq \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \geq \ 0,$$

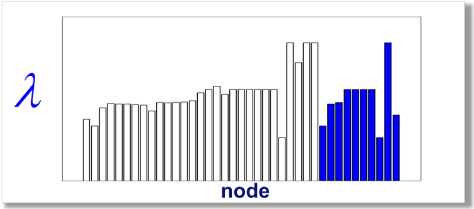
$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

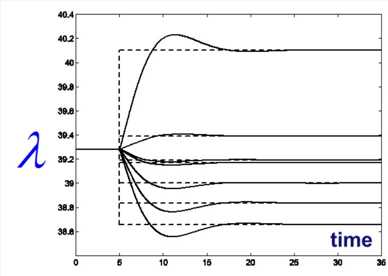


max-based complementarity integrator

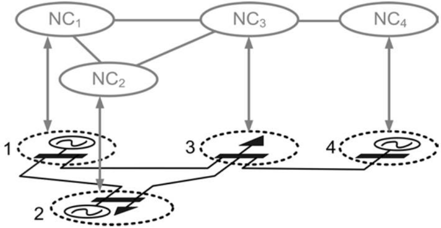

P_L
G_1
G_2
G_3
G_4
G_5
G_6
G_7
G_8
G_9
G_{10}
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 35 36 38 39


P_L
2.2
2
1.8
1.6
1.4
0 5 10 15 20 25 30 35
time


λ
node


λ
40.4
40.2
40
39.8
39.6
39.4
39.2
39
38.8
38.6
0 5 10 15 20 25 30 35
time

Distributed, real-time, price-based control

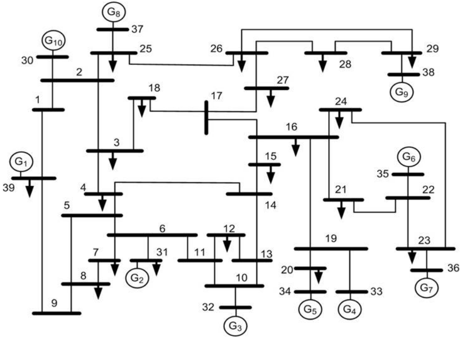


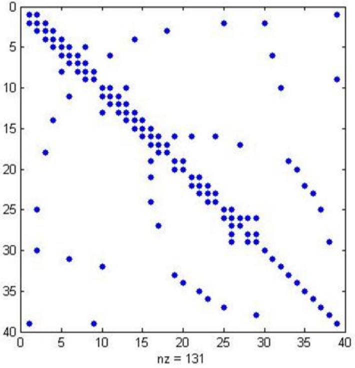
$B\lambda + L^\top \mu = 0$, λ prices for local balance, μ prices for not overloanding the lines

$$\left(\begin{array}{cccc|cc} b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} & b_{13} \\ -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} & 0 \\ -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 & -b_{13} \\ 0 & 0 & -b_{34} & b_{34} & 0 & 0 \end{array}\right) \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \hline \mu_{12} \\ \mu_{13} \end{pmatrix} = 0,$$

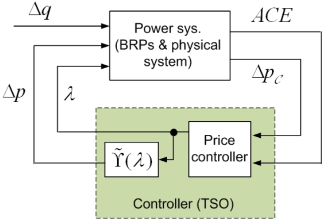
Distributed, real-time, price-based control

$$B\lambda + L^{\top}\mu = 0$$





Distributed, real-time, price-based control



Optimality conditions

$$\begin{aligned} \beta(p^\star) - \lambda^\star &= 0 \\ p^\star - B\theta^\star &= 0 \\ B\lambda^\star + L^\top \mu^\star &= 0 \\ 0 \le (-L\theta^\star + \bar{e}_\mathcal{E}) \quad \perp \quad \mu^\star &\ge 0 \end{aligned}$$

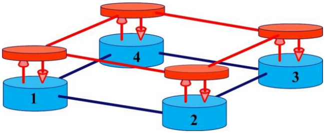
Real-time nodal price based SC controller (each control area balanced separately)

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_C \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix},$$

$$0 \le w \;\perp\; K_0 x_\mu + \Delta p_C + w \ge 0,$$

$$\lambda = \begin{pmatrix} \boxed{I} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \tilde{\Upsilon}(\lambda)$$

Distributed, real-time, price-based control



Optimality conditions

$$\begin{aligned} \beta(p^\star) - \lambda^\star &= 0 \\ p^\star - B\theta^\star &= 0 \\ B\lambda^\star + L^\top \mu^\star &= 0 \\ 0 \leq (-L\theta^\star + \bar{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star &\geq 0 \end{aligned}$$

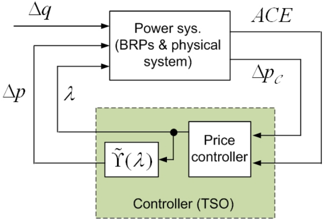
Real-time nodal price based SC controller (each control area balanced separately)

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix},$$

$$0 \leq w \ \perp \ K_0 x_\mu + \Delta p_{\mathcal{C}} + w \geq 0,$$

$$\lambda = \begin{pmatrix} \boxed{I} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \tilde{\Upsilon}(\lambda)$$

Distributed, real-time, price-based control



Optimality conditions

$$\begin{aligned} \beta(p^\star) - \lambda^\star &= 0 \\ p^\star - B\theta^\star &= 0 \\ B\lambda^\star + L^\top \mu^\star &= 0 \\ 0 \le (-L\theta^\star + \bar{e}_\mathcal{E}) \quad \perp \quad \mu^\star &\ge 0 \end{aligned}$$

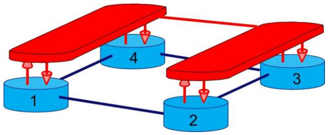
Real-time zonal price based SC controller (each control area balanced separately)

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta pc \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix}$$

$$0 \le w \ \perp \ K_0 x_\mu + \Delta p_\mathcal{C} + w \ge 0$$

$$\lambda_\mathcal{Z} = \begin{pmatrix} \boxed{F(\cdot)} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \Upsilon(\lambda_\mathcal{Z})$$

Distributed, real-time, price-based control



Optimality conditions

$$\begin{aligned} \beta(p^\star) - \lambda^\star &= 0 \\ p^\star - B\theta^\star &= 0 \\ B\lambda^\star + L^\top \mu^\star &= 0 \\ 0 \le (-L\theta^\star + \bar{e}_{\mathcal{E}}) \quad \perp& \quad \mu^\star \ge 0 \end{aligned}$$

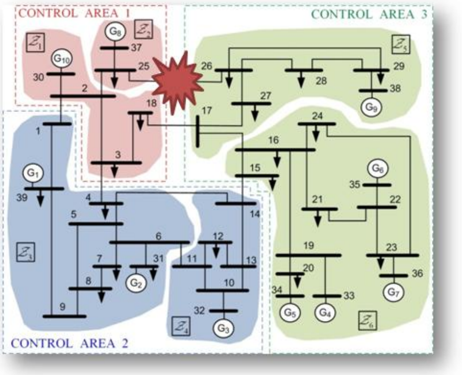
Real-time zonal price based SC controller (each control area balanced separately)

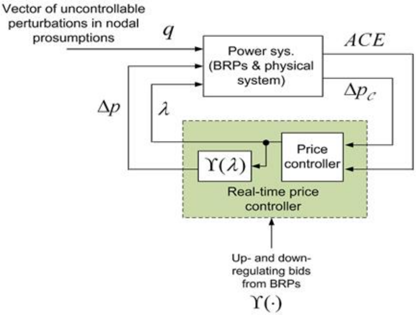
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta pc \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix}$$

$$0 \le w \ \perp \ K_0 x_\mu + \Delta p_{\mathcal{C}} + w \ge 0$$

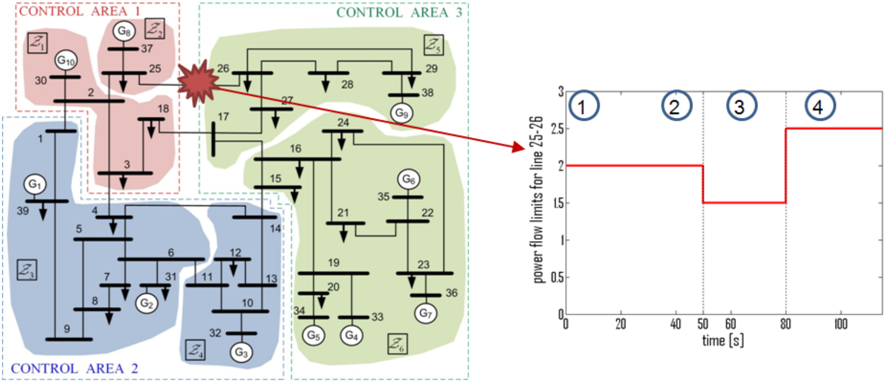
$$\lambda_{\mathcal{Z}} = \begin{pmatrix} \boxed{F(\cdot)} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \Upsilon(\lambda_{\mathcal{Z}})$$

Distributed, real-time, price-based congestion control

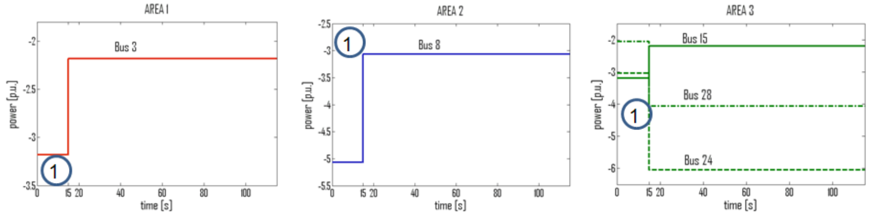




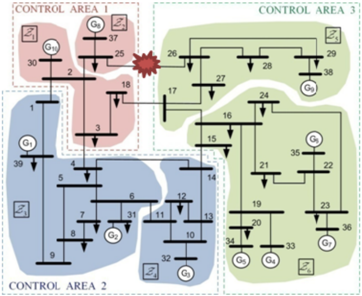
EXAMPLE

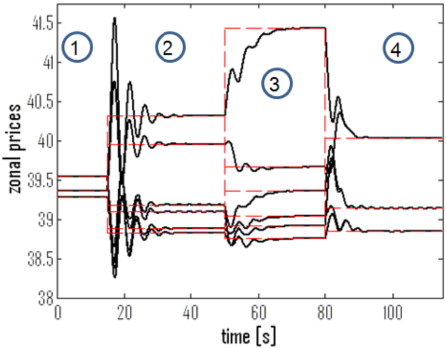


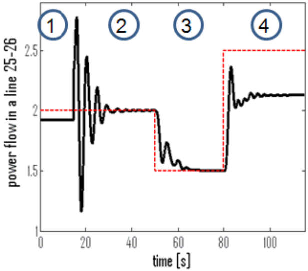
CONTROL AREA 1
CONTROL AREA 2
CONTROL AREA 3
power flow limits for line 25-26
time [s]
1
2
3
4

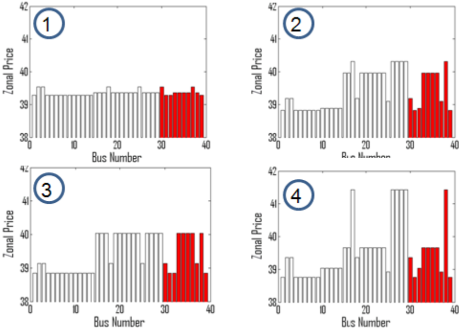


AREA 1
Bus 3
AREA 2
Bus 8
AREA 3
Bus 15
Bus 28
Bus 24
power [p.u.]
time [s]
1


CONTROL AREA 1
CONTROL AREA 2
CONTROL AREA 3


zonal prices
time [s]
1
2
3
4


power flow in a line 25-26
time [s]
1
2
3
4


Zonal Price
Bus Number
1
2
3
4

Outline

1 Power system basics

2 Market-based operation (STATIC)

3 Congestion management (STATIC)

4 Market-based robust spatial distribution of ancillary services (STATIC + DYNAMIC)

5 Closed loop price-based operation (DYNAMIC)
- Optimization as feedback controller
- Price-based control of power systems

6 Conclusions

Conclusions and messages

- "smart? $=$ hidden $+$ invisible"
- make it work $\rightarrow$ make it big $\rightarrow$ make it sustainable (massive automation)
- extremly complex system (many NP-hard problems, large-scale,...) - yet it works very robustly
- Today's robustness (work $+$ big): physics on our side $+$ partly due to conservative engineering
- Future: changes in physical layer $+$ smart? (massive automation) $\rightarrow$ increased complexity. Robustness (fragility?), efficiency, scalability?
- think in terms of: arhiteicture (layering), modules and protocols
- Optimization (duality!): holistic approach to market (and control)
- Huge area for important research (exciting parallel research in control systems field)

UNIJE (smart island Unije): sustainable island

http://insulae-h2020.eu/ https://insulae.wp.fsb.hr/