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Mixed control of vibrational systems

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Setting

Linear vibrational system, modeled as a 2nd order matrix differential equation

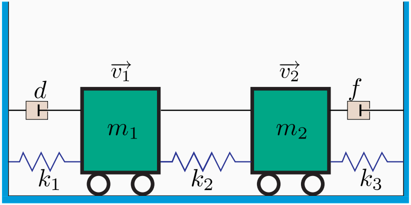
$$M\ddot{x} + C\dot{x} + Kx = F$$

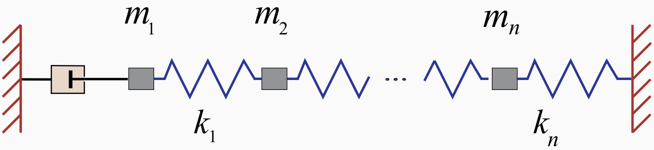
M mass matrix

C damping matrix

K stiffness matrix

F external force





Aim

Attenuate unwanted vibrations of the system by the use of passive damping.

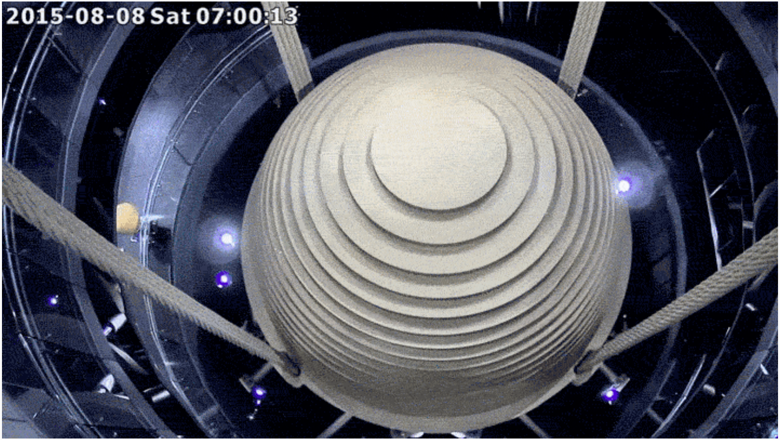
In other words, find an appropriate damping matrix C such that the system vibrates as little as possible.

System will have N modes of vibration, N dimension of the system, not all modes are dangerous. Usually there is a range of dangerous ones.

Why?

Why?

Does it work?



Can one succesfully attenuate dangerous vibration by passive damping?
Yes!

How to do it?

Pose the problem of find good/optimal damping mechanism as an optimization problem. So we need an optimization criterion.

(Too) many criteria in use for different applications (Google Scholar has 342 000 hits for ”damping criteria”).

How to do it?

Important classes:

➔ based on the analysis of stationary system (external force $F = 0$, excitation by the initial condition), some interesting ones:

- $\rightarrow$ based on eigenvalues (e.g. max $\Re\lambda$, max $\frac{\Re\lambda}{|\lambda|}$)
- $\rightarrow$ based on the total energy (e.g. max $\int_0^\infty E(t)\mathrm{d}t$, avg. $\int_0^\infty E(t)\mathrm{d}t$)

➔ based on the analysis of excitation by a particular external force

- $\rightarrow$ harmonic excitation
- $\rightarrow$ periodic non-harmonic excitation

How to do it?

For a random external force we can use the machinery of control theory.
Again different criteria, most usefull ones:

- ➔ H_2 norm
- ➔ H_∞ norm

H_2 norm criterion: external force modeled by (white/coloured) noise, we obtain best damping for a ”typical” external force.
H_2 norm criterion seems like the best choice for a large class of vibrational systems (non-critical systems, where external environment changes).

Problem

When doing numerics, for some configurations we obtain that the damping coefficient should be as large as possible.

This does not make any sense.

It is not a numerics glitch. Can be shown analytically. What is going on?

Toy (academic) example

Let's say we can choose any damping matrix D we want.

What is the best damping matrix for the H_2 norm criterion?

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We are missing the initial condition, control theory setting does not see it.


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If you pour concrete over your structure, it surely will not vibrate.

Solution

Use a mixed criterion, for example

$$\text{avg.} \int_0^\infty E(t)\mathrm{d}t \quad + \quad \|\cdot\|_{H_2}$$

We mix one criterion which depends on the initial condition but for the stationary problem and one control-theoretic which does not depend on the initial condition but treats a class of interesting external forces.
Many options, another one:

$$\max \text{ total energy } + \quad H_\infty \text{ norm }.$$

Does it work?

It is well known that the best global damping matrix for a wide class of criteria is given by a modal damping matrix. These are matrices for which the vibrational system decomposes to a system of uncoupled 1 DOF systems.

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Numerical expariments confirm that mixed criteria lead to physically meaningful damping matrices.

Thanks for the attention!

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