Interpolation-based parametric model reduction for efficient damping optimization

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Hrvatska zaklada za znanost

[ROME 2017]

22 Jun 2017

We consider vibrational system

$$M\ddot{q}(t) + \overbrace{\left(C_{int} + B_2 G B_2^T\right)}^{\text{C=damping part}}\dot{q}(t) + Kq(t) = E_2 w(t),$$
$$z(t) = H_1 q(t).$$

- $M, K > 0$ mass and stiffness,
- E_2 primary excitation matrix,
- C_{int} internal damping e.g. $C_{int} = \alpha_c C_{crit}$, where $C_{crit} = 2M^{1/2}\sqrt{M^{-1/2}KM^{-1/2}}M^{1/2}$,
- $G = \operatorname{diag}(g_1, g_2, \dots, g_p)$, $g_i \geq 0$ represents coefficients of damper,
- q state vector and z is output vector determined by H_1,
- vector w corresponds to primary excitation input.

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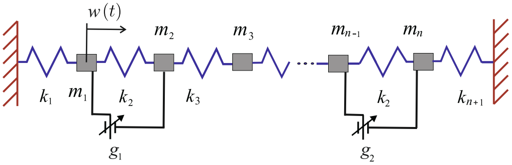
Example: *n-mass oscillator or oscillator ladder*



$$M = \mathrm{diag}(m_1, m_2, \ldots, m_n), \quad C = B_2 G B_2^T + \alpha_c C_{crit},$$

$$B_2 G B_2^T = g_1(e_k - e_{k+1})(e_k - e_{k+1})^T + g_2(e_j - e_{j+1})(e_j - e_{j+1})^T.$$

$$K = \begin{pmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & k_2 + k_3 & -k_3 & & \\ & \ddots & \ddots & \ddots & \\ & & -k_{n-1} & k_{n-1} + k_n & -k_n \\ & & & -k_n & k_n + k_{n+1} \end{pmatrix}.$$

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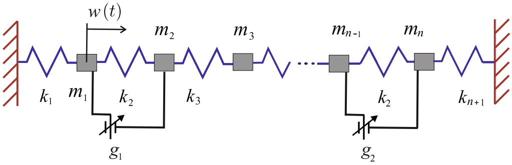

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Linearization

- Using Φ which simultaneously diagonalizes M and K

$$\Phi^T K \Phi = \Omega^2 = \operatorname{diag}(\omega_1^2, \ldots, \omega_n^2) \quad \text{and} \quad \Phi^T M \Phi = I,$$

$0 < \omega_1 \leq \omega_2 \leq \ldots \leq \omega_n.$

- with $\hat{x}_1 = \Omega\Phi^{-1}q(t)$ and $\hat{x}_2 = \Phi^{-1}\dot{q}(t)$ system can be written as :

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \begin{bmatrix} 0 \\ \Phi^T E_2 \end{bmatrix} w(t),$$

$$z(t) = \begin{bmatrix} H_1\Phi\Omega^{-1} & 0 \end{bmatrix} x(t), \quad \text{where}$$

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & \Omega \\ -\Omega & -\alpha\Omega - \Phi^T B_2 G B_2^T \Phi \end{bmatrix}.$$

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Problem :

Determine "optimal" damping matrix C which will minimize the effect of the input w on the output z.

For criterion one can use e.g.:

- $\mathcal{H}_2$ norm of a system

$$\|H\|_{\mathcal{H}_2} = \left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} \operatorname{tr}\left(H(j\omega)^* H(j\omega) \right) d\omega \right)^{\frac{1}{2}}$$

transfer function $H(s) = H_1(s^2 M + sC + K)^{-1} E_2, \quad s \in \mathbb{C}.$

- $\mathcal{H}_\infty$ norm of a system

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Optimization problem for $\mathcal{H}_2$ norm

Impulse response energy leads to

$$\hat{A}^T \hat{X} + \hat{X}\hat{A} = -\hat{H}^T \hat{H},$$

$$\hat{A} = \begin{bmatrix} 0 & \Omega \\ -\Omega & -\alpha\Omega - \Phi^T B_2 G B_2^T \Phi \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} H_1 \Phi \Omega^{-1} & 0 \end{bmatrix},$$

With $\hat{X} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$, it holds

$$J_2 = \mathrm{tr}\,(E_2^T \Phi X_{22} \Phi^T E_2) \quad \to \min.$$

Model order reduction: provides efficient approximation of energy J_2 needed for optimization of G.

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Model order reduction

With $W_r \in \mathbb{R}^{n\times r}$, $q(t) = W_r q_r(t)$ and $V_r \in \mathbb{R}^{n\times r}$ we obtain reduced system

$$M_r \ddot{q}_r(t) + C_r \dot{q}_r(t) + K_r q_r(t) = E_r w(t) \quad \text{where}$$
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Transfer function of reduced system is

$$H_r(s) = H_r(s^2 M_r + sC_r + K_r)^{-1} E_r, \quad s \in \mathbb{C}.$$

We choose W_r and V_r to enforce (tangential) interpolation.

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Model reduction by Interpolation

For selected points $\sigma_1, \sigma_2, \ldots, \sigma_r \in \mathbb{C}$ and directions $b_1, \ldots, b_r$ and $c_1, \ldots, c_r$ find $H_r(s)$ such that

$$c_i^T H(\sigma_i) = c_i^T H_r(\sigma_i)$$
$$H(\sigma_i) b_i = H_r(\sigma_i) b_i$$
$$c_i^T H'(\sigma_i) b_i = c_i^T H_r'(\sigma_i) b_i.$$

Moreover: we would like to have an approximation s.t. $\|\cdot\|_{\mathcal{H}_2}$ is optimally approximated, i.e. find local minimizer for $\|H - H_r\|_{\mathcal{H}_2}$.

This can be done in general framework efficiently[1] using structure preserving Model Reduction.
Additionally to preserve structure in second order system: $V_r = W_r$.

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Alg. 1: Iterative rational Krylov algorithm (IRKA)

Require: System matrices defining, and given gains $g_1, g_2, \dots, g_p$, initial shift selection $\sigma_1 \dots, \sigma_r$; initial tangent directions $r_1, \dots, r_r$.

$V_r = [(\sigma_1^2 M + \sigma_1 C + K)^{-1} B r_1, \dots, (\sigma_r^2 M + \sigma_r C + K)^{-1} B r_r]$;

for $j = 1, \dots, \max_{it}$ **do**

  Form reduced system determined by: $M_r = V_r^* M V_r$, $C_r = V_r^* C V_r$, $K_r = V_r^* K V_r$, $E_r = V_r^* E_2$, $H_r = H_1 V_r$

  Consider quadratic eigenvalue problem $(M_r \lambda_i^2 + C_r \lambda_i + K_r) x_i = 0$, $x_i \neq 0$, $i = 1, \dots, 2r$ and reduce system to r states in order to have r eigenvalues $\tilde{\lambda}_1, \dots, \tilde{\lambda}_r$ closed under conjugation

  $\sigma_i = -\tilde{\lambda}_i$ and update r_i, $i = 1, \dots, r$

  $V_r = [(\sigma_1^2 M + \sigma_1 C + K)^{-1} B r_1, \dots, (\sigma_r^2 M + \sigma_r C + K)^{-1} B r_r]$;

  if converged **then**

    return $V = V_r$

  end if

end for

Usage of modal coordinates

For given shift σ and direction r_i in solving

$$(\sigma^2 M + \sigma_1 C + K)^{-1} B r_i$$

we apply reduction directly to system in modal coordinates.
Here we use Sherman-Morrison-Woodbury formula:

$$(\sigma^2 I + \sigma\alpha\Omega + \sigma\Phi^T B_2 G B_2^T \Phi + \Omega^2)^{-1}\Phi^T B r_i = T^{-1}\Phi^T B r_i - sT^{-1}B_g(I_p + sB_g^T T^{-1} B_g)^{-1} B_g^T T^{-1}\Phi^T B r_i$$

where

$$T = (\sigma^2 I + \sigma\alpha\Omega + \Omega^2),$$
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Internal reduction

a) internal reduction based on balanced truncation;
used balanced truncation method applied to the linearized model.

b) internal reduction based on IRKA algorithm;
apply additional reduction using IRKA approach to linearized model.

c) internal reduction based on dominant poles;
Since the transfer function

$$F(s) = \sum_{i=1}^{2n} \frac{R_i}{s - \lambda_i} \quad \text{with} \quad R_i = (H_1 x_i)(y_i^* E_2)\lambda_i,$$

where $\lambda_i \in \mathbb{C}$, $x_i,\ y_i$ are, respectively, eigenvalues, right and left eigenvectors of the QEP.
We maintain the r poles with the largest values of $\frac{\|R_i\|}{|\mathrm{Re}\,(\lambda_i)|}$

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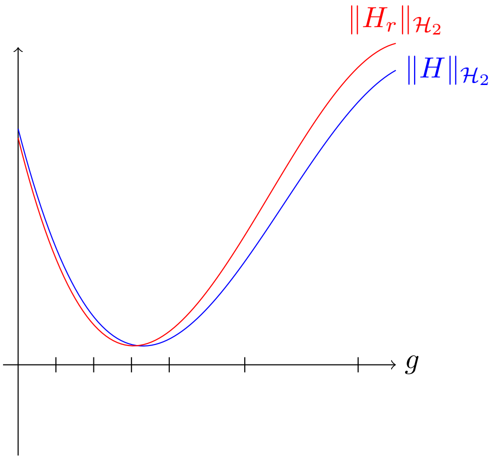
We propose the following sampling strategies

- fixed sampling parameters (depending on parameter feasible area);
- adaptive sampling during optimization.



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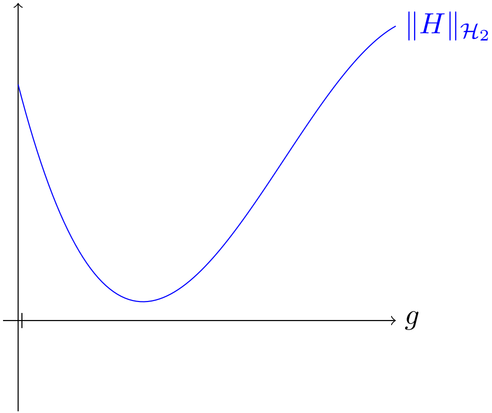

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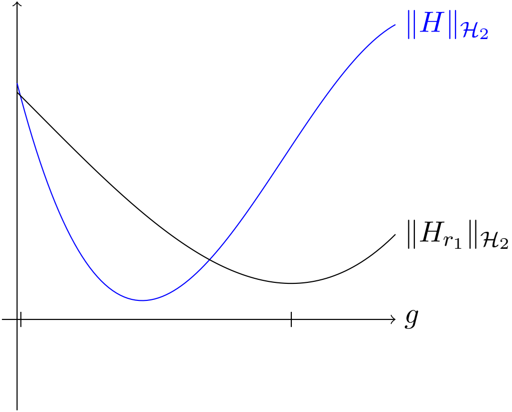

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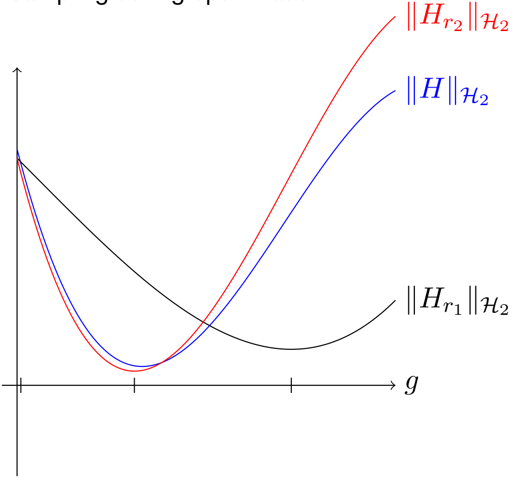

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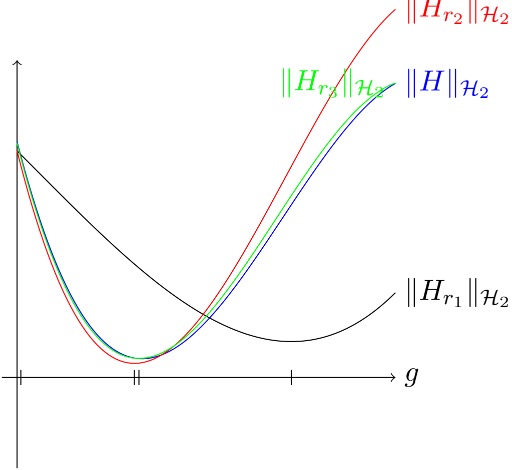

Optimization approach using fixed sampling parameters

Algorithm 2:

Require: System matrices; initial value for shift selection $\sigma_1 \ldots, \sigma_r$ and initial directions $r_1, \ldots, r_r$;
number of wanted poles k_{wanted} for each setting of parameters.;
set of sampling parameters $g^1, \ldots, g^m$.

Ensure: Approximate optimal gains.

1: **for** $j = 1, \ldots, m$ **do**
2: With IRKA Algorithm using gain g^i calculate V^i
3: **end for**
4: $X = \text{orth}([V^1, \ldots, V^m])$.
5: Form a reduced system using X .
6: Find an optimal gains by using an appropriate optimization procedure on obtained reduced system.

Optimization approach using adaptive sampling

Algorithm 3:

Require: System matrices; initial value for shift selection $\sigma_1 \ldots, \sigma_r$ and initial directions $r_1, \ldots, r_r$;
number of wanted poles k_{wanted} for each setting of parameters.;
the first sampling parameters g^0.

Ensure: Approximate optimal gains.

1: $j = 0$;
2: **repeat**
3: With IRKA Algorithm using gain g^j calculate V^j
4: Form reduced system using $X = \mathrm{orth}([V^0, V^1, \ldots, V^j])$.
5: $j = j + 1$
6: Find an approximation of optimal gains by using obtained reduced system and denote it by g^j.
7: **until** $|g^j - g^{j-1}| < tol_g$
8: return g^j

Parametric dominant pole algorithm

Reduced system is obtained with $q(t) = Xq_k(t)$ where $X \in \mathbb{C}^{n \times k}$ span the eigenspaces associated with the k dominant poles, efficiently calculated.

For initial parameters $g^{(1)} = 0, g^{(2)}, \ldots, g^{(m)}$ we merge together corresponding right eigenspaces $X^{(j)}$. $\rightsquigarrow$ for initial parameters the original and reduced model have similar behavior near dominant poles.

Determination of sampling parameters $g^{(2)}, \ldots, g^{(m)}$: adaptively, depending on residual error bound.

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For initial parameters $g^{(1)} = 0, g^{(2)}, \ldots, g^{(m)}$ we merge together corresponding right eigenspaces $X^{(j)}$. $\rightsquigarrow$ for initial parameters the original and reduced model have similar behavior near dominant poles.

Determination of sampling parameters $g^{(2)}, \ldots, g^{(m)}$: adaptively, depending on residual error bound.

[2] P. Benner, P. Kürschner, Z. Tomljanović, N. Truhar, Semi-active damping optimization of vibrational systems using the parametric dominant pole algorithm, Journal of Applied Mathematics and Mechanics, 2016

We will consider the n-mass oscillator with 1900 masses where

$$M = \operatorname{diag}(m_1, m_2, \dots, m_n),$$

$$K = \begin{pmatrix} 2k_1 + 2k_2 & -k_2 & -k_3 & & & \\ -k_2 & 2k_2 + 2k_3 & -k_3 & -k_4 & & \\ -k_3 & -k_3 & 2k_3 + 2k_4 & -k_4 & -k_5 & \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}.$$

With configuration:

$$n = 1900; \quad \alpha_c = 0.005$$

$$k_i = 500, \quad \forall i; \qquad m_i = \begin{cases} 144 - \frac{3}{20} i, & i = 1, \dots, 475, \\ \frac{i}{10} + 25, & i = 476, \dots, 1900. \end{cases}$$

Primary excitation matrix is applied to 10 consecutive masses, i.e.

$$E_2(471:481, 1:10) = \mathrm{diag}(10, 20, 30, 40, 50, 50, 40, 30, 20, 10),$$

We are interested in the 18 states equally distributed

$$H_1(1:18, 100:100:1800) = I_{18\times 18}$$

The geometry of external damping is determined by four dampers with

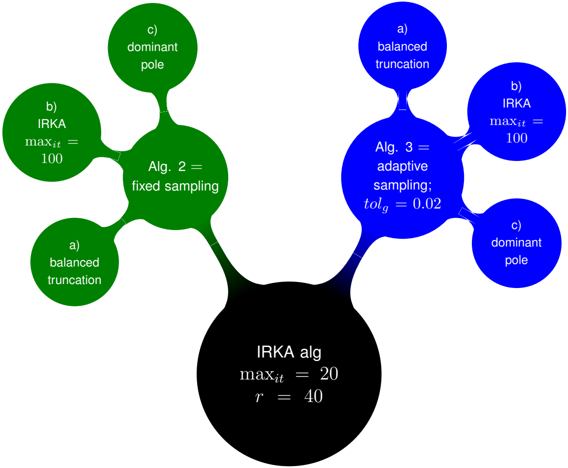
$$B_2 = [e_i \; e_{i+1} \; e_k \; e_{k+1}]\,,$$

for 44 different damping configurations (i, k), where

$$G = \mathrm{diag}\,(g_1, g_1, g_2, g_2).$$

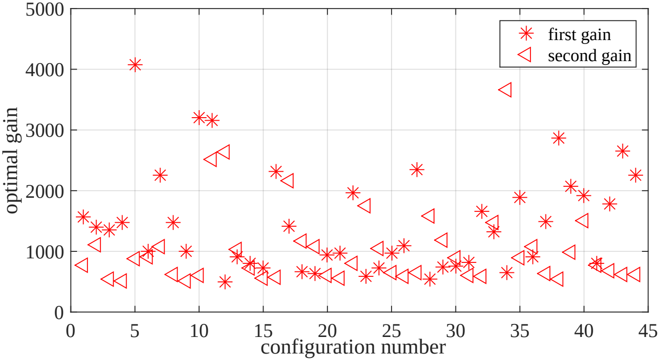
Comparison:

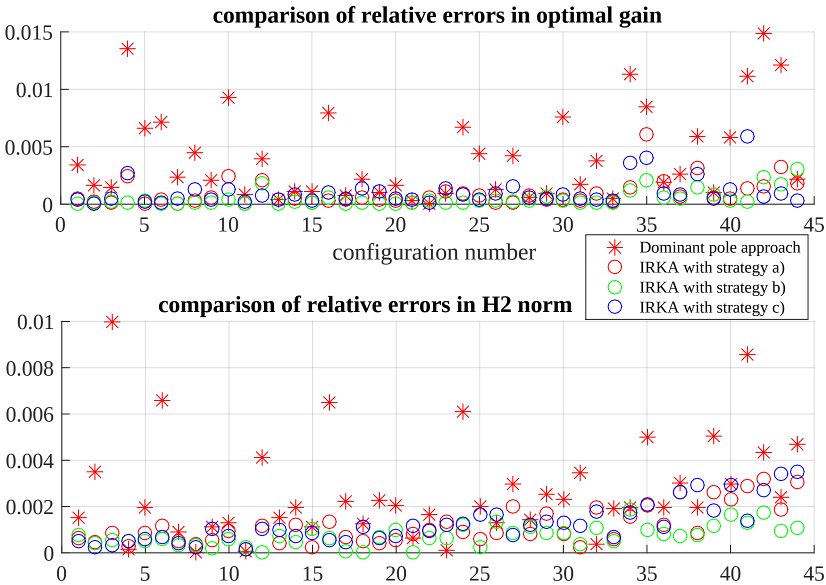
- IRKA with fixed sampling $\leftrightarrow$ parametric dominant pole algorithm,
- IRKA with fixed sampling $\leftrightarrow$ IRKA with adaptive sampling.



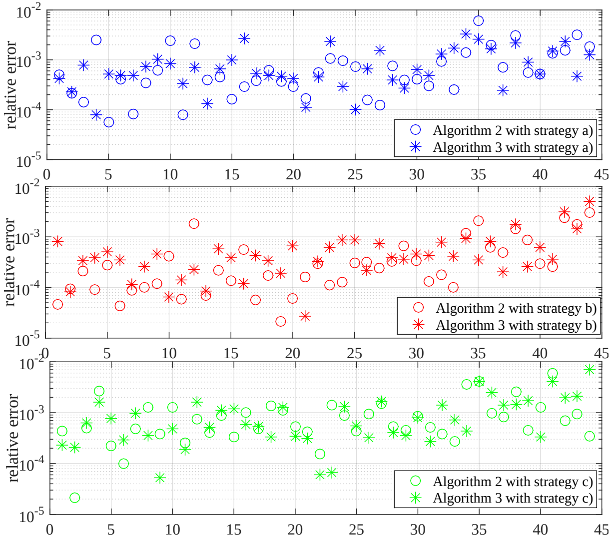
Gains for fixed sampling

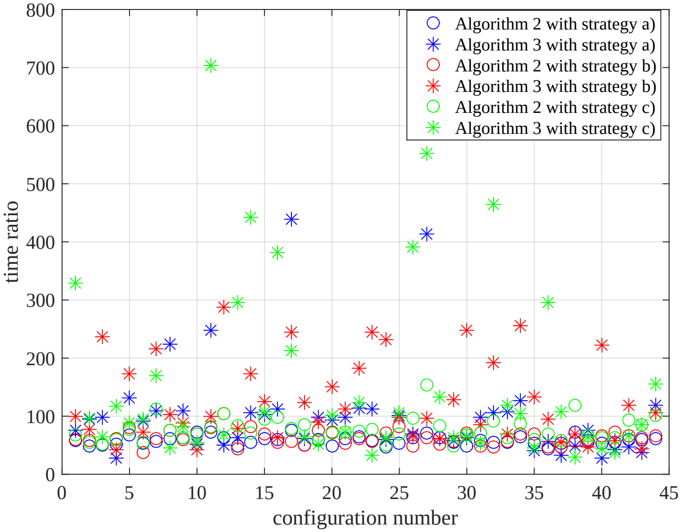
$$g^{(0)} = (0,\, 0) \quad \Rightarrow \quad g^{(1)}$$
$$g^{(2)} = (1000,\, 1000)$$
$$g^{(3)} = (100,\, 1000)$$
$$g^{(4)} = (1000,\, 100)$$




comparison of relative errors in optimal gain
0.015
0.01
0.005
0
0
5
10
15
20
25
30
35
40
45
configuration number
Dominant pole approach
IRKA with strategy a)
IRKA with strategy b)
IRKA with strategy c)
comparison of relative errors in H2 norm
0.01
0.008
0.006
0.004
0.002
0

Comparison of relative errors in optimal gain




Algorithm 2 with strategy a)
Algorithm 3 with strategy a)
Algorithm 2 with strategy b)
Algorithm 3 with strategy b)
Algorithm 2 with strategy c)
Algorithm 3 with strategy c)
time ratio
configuration number
0
100
200
300
400
500
600
700
800
0
5
10
15
20
25
30
35
40
45

Thank you for your attention!